

This article appeared in a journal published by Elsevier. The attached copy is furnished to the author for internal non-commercial research and education use, including for instruction at the authors institution and sharing with colleagues.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/copyright>



# The long swings in the spot exchange rates and the complex unit roots hypothesis

Haitham A. Al-Zoubi \*

*Associate Professor of Finance, Department of Economics & Finance,  
 United Arab Emirates University, Al-Ain, United Arab Emirates*

Received 6 January 2006; accepted 17 October 2006

Available online 27 November 2006

---

## Abstract

This paper addresses whether the spot exchange rates display long swings and whether these swings are persistent. The null from the naïve random walk theory is that they do not: if they would be unit roots with positive drifts they would converge to infinity. However, if they would be driftless unit roots they would assign negative values, which is unrealistic. We test this by examining whether the yearly changes of spot exchange rates display complex conjugate unit roots against the stationary hypothesis. We reject the hypothesis that the yearly changes in exchange rates are stationary in favor of cyclical, complex unit roots. The periodogram based cycle duration analysis reveals that the long swings in the exchange rates are persistent.

© 2006 Elsevier B.V. All rights reserved.

*JEL classification:* C5; F3; G1

*Keywords:* Complex unit roots; Foreign exchange rates; Long swings

---

## 1. Introduction

The difficulty in modeling the nominal and real exchange rates has been a longstanding puzzle in international finance. The overshooting theory of exchange rates determination suggests that exchange rates tend to deviate from their fundamental values in the short-run and as time passes the processes return to its long-run equilibrium.<sup>1</sup> However, early tests of exchange rates behavior, as documented by Roll (1979), Frankel (1981) and Hakkio (1986) suggest that exchange rates

---

\* Tel.: +971 50 138 3553.

*E-mail address:* [halzoubi@hu.edu.jo](mailto:halzoubi@hu.edu.jo).

<sup>1</sup> The Dornbusch (1976) model of exchange rates determination suggests that commodities prices adjust slowly and the nominal exchange rate adjusts quickly to a monetary shock; as a result, the nominal exchange rates are stationary.

fluctuations are driven by permanent components. This entails that deviations from purchasing power parity (PPP) are cumulative and not eventually self-reverting (see Meese and Rogoff, 1988).

In the last decade, however, there was significant evidence that the real exchange rates do not behave like a random walk but as a stationary process.<sup>2</sup> Using the adjusted *t*-test proposed by Levin and Lin (1992), Oh (1996) and Lothian (1997) provide evidence supporting PPP. Choi (2001) and Papell and Theodoridis (1998) use panel unit root tests to get the same conclusion. Bergman and Hanson (2005) provide evidence that the real exchange rate among major currencies can be described by a stationary, two-state Markov switching process. In contrast, Engel (2000, p. 244) strongly contradicts this view; he argues. “However, this recent literature may have reached the wrong conclusion. Cochrane (1991), Blough (1992) and Faust (1996) contend that there is always a non-stationary representation for a time series that is arbitrarily close to any stationary representation.”

According to Cochrane–Blough–Faust econometric theory it is impossible to differentiate between the case in which the random walk component of asset price has a uninformed small innovation variance (in which case, the asset price is non-stationary), and the case of a zero innovation variance for the random walk component (in which case, the random walk component is constant and the asset price is stationary.) A test which rejects a unit root in asset prices when the random walk component has a uninformed small innovation variance has a tendency to reject the unit root behavior in asset price with the same incidence for the case in which the random walk component is constant. However, tests with large powers to reject the null of real-valued unit root (equal to one), when the random walk component has a uninformed small innovation variance, must have large size distortions against the null that the random walk component is constant.

In the corresponding literature of nominal exchange rates, the naïve random walk specification does not deter the profession from exploring predictable patterns in exchange rates movement and contradicting possible interactions between spot exchange rates and their fundamentals. Among the recent attempts, Engel and Hamilton (1990) introduce the long swings model of the exchange rates where the spot rates tend to move in one direction for long period of time.<sup>3</sup> They reject the null hypothesis that the exchange rates display a random walk in favor of a sequence of stochastic, segmented time trends. The evidence supporting long swings is also reported by Bollen et al. (2000), Dewachter (2001), and Bazdresch and Werner (2005).<sup>4</sup>

Nevertheless, even with the benefit of 15 years of hindsight, the Cochrane–Blough–Faust–Engel results have not been convincingly highlighted: evidences that exchange rate is stationary (e.g., Bergman and Hanson, 2005; Taylor, 2002; Mark, 1995) still rely on unit root tests that have large size distortions or stationary tests that have very low power.

In this paper, we show that the long swing’s behavior of the exchange rates suggested by Engel and Hamilton (1990) is due mainly to complex-conjugate unit roots in the autoregressive lag polynomial of the spot exchange rates. As is well known, autoregressive (AR) processes with complex unit roots are non-stationary, and are in fact more appealing than AR processes with a single unit root equal to one, because these processes exhibit a persistent cyclical behavior. Cyclical output patterns could translate into exchange rate cycles, since the international transmission of

---

<sup>2</sup> The early evidence of stationary real exchange rates is related to Frankel (1986), Huizinga (1987) and Abuaf and Jorion (1990).

<sup>3</sup> Engel (1994) extends the exercise to cover 18 spot rates and suggests that the long swings model outperforms the random walk model in both in-sample and out-of-sample forecasts.

<sup>4</sup> Engel and Hamilton (1990), Bollen et al. (2000) and Dewachter (2001) use the Markov switching model that allows the exchange rate dynamics to interchange between two regimes.

the business cycle traditionally takes place through the trade balance (e.g., Williamson and Milner, 1991).<sup>5</sup> This result notwithstanding, the nominal exchange rates show a long, cyclical patterns, not dissimilar to a business cycle patterns.

The statistical properties of the tests that we use – which rely on complex unit roots – have large local power with low size distortions in comparison with regular tests that rely on real valued unit root (see Cochrane, 1991; Blough, 1992; Faust, 1996; Engel, 2000; Bierens, 2001). The standardized periodogram that we utilize allows testing for duration stabilization and long swings in the exchange rates without setting the number of regimes *a priori*.<sup>6</sup> The null hypothesis is constructed prior to looking at the data generating process where selected periodogram frequencies could not be correspondent to the monetary policy cycles. This is a kin to pretesting since it provides lower size biases (see Bierens, 2001).

The paper proceeds as follows: Section 2 describes the data and develops the hypothesis to be tested. Section 3, tests the hypotheses and explain the results. Section 4 concludes.

## 2. Data and methodology

### 2.1. Data

We consider the end-of-the month spot exchange rates that cover the period spans from July 1973 to December 2002. The transformed data are against the U.S. dollar, obtained from Data Research Incorporate (DRI) database, for the Canadian dollar, the Swiss franc, and the British pound. We use monthly observations to provide high frequencies for cycle durations as in Bierens (2001). Since we are interested in examining complex-conjugate unit roots in exchange rates rather than real-valued unit root or seasonal unit roots, we transform the data to the annual changes on log levels (1-year overlapping holding period returns). The transformed series are graphed in Fig. 1.

### 2.2. Methodology

To motivate the estimand given below, we begin by considering the standardized continuous periodogram of time series  $y_t$ ,  $t = 1, \dots, n$ ,

$$\rho(\xi) = \frac{2}{n\sigma_y^2} \left( \left( \sum_{t=1}^n y_t \cos(\varepsilon_t) \right)^2 + \left( \sum_{t=1}^n y_t \sin(\varepsilon_t) \right)^2 \right), \quad (1)$$

<sup>5</sup> Movements in exchange rates are often tied to changes in business cycle conditions. A common feature of nearly all models of exchange rates is the prediction that the real exchange rate between two countries is correlated with the ratio of business cycle conditions in the two countries. In the real business cycle framework, some suggests that technology shocks are the driven forces behind the relation (e.g., Lucas, 1982; Svensson, 1985; Stockman, 1990; Stockman and Tesar, 1995; Obstfeld and Rogoff, 1995; Kollmann, 1997). The basic rationally is that as domestic output rises relative to foreign output exogenously, real exchange rate depreciates because domestic price level declines relative to its foreign counterparts. Movements of real exchange rates are often tied to movements in nominal exchange rates. For example, Rogoff (2002) finds that sticky prices and rapid clearing financial markets force adjustment onto the nominal exchange rates. The equilibrium exchange rate models of Lucas (1982) and Stockman (1987) predict that real exchange rate changes cause nominal exchange rate changes. As in Dornbusch (1976) and Chari et al. (1998), the shock to aggregate demand reduces the home nominal interest rate relative to the foreign rate, requiring a short-run depreciation in excess of the long-run depreciation.

<sup>6</sup> Dacco and Satchell (1999) conclude that the forecast performance of Markov switching models is very sensitive to regimes misclassification.

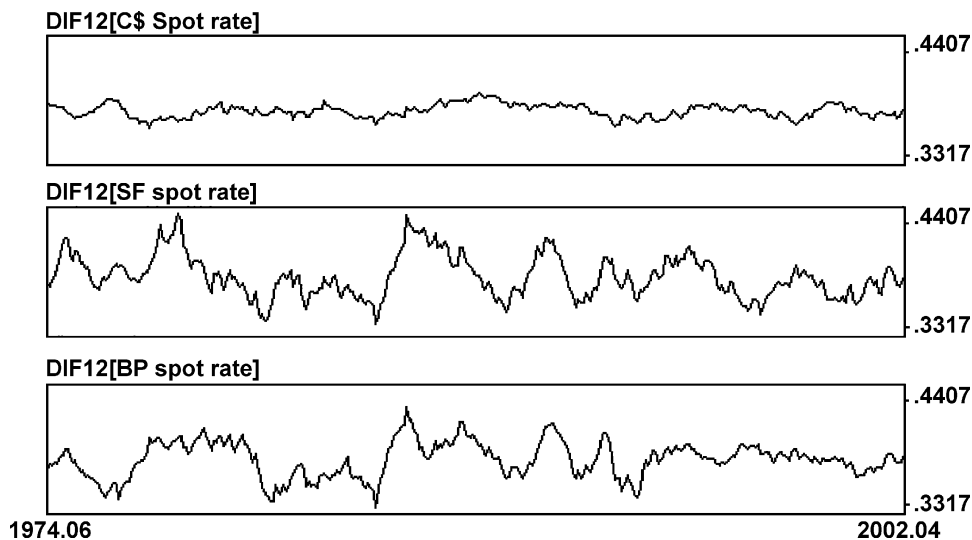


Fig. 1. The yearly changes of the spot exchange rates of the Canadian dollar, the Swiss franc, and the British pound, respectively.

where  $\varepsilon$  is a disturbance term with zero mean and variance  $\sigma_\varepsilon^2$ ,  $\sigma_y^2$  is the sample variance and  $\xi$  is a random function in  $(0, \pi)$  that is displayed for  $2\pi/k$ ,  $k=2, \dots, n$ . So that  $k$  corresponds to possible cycle period.

As in Bierens (2001) and Diaz-Emparanza (2004), we are interested in testing the hypothesis that  $y_t$  displays complex unit roots of a form:

$$\prod_{j=1}^k (1 - 2\cos(\phi_j)L + L^2)y_t = \mu + \eta(L)\varepsilon_t, \tag{2}$$

where  $(\phi_j) \in (0, 2\pi) - \{\pi\}$ ,  $\varepsilon_t$  is an *i.i.d*  $(0, 1)$  process,  $L$  the lag operator, and  $\eta(L)$  is the lag polynomial with roots outside complex circle of one. Note that we consider the case suggested by Diaz-Emparanza (2004) where  $(\phi_j) \in (0, 2\pi) - \{\pi\}$  to take into consideration the cycle caused by the root  $\cos(\phi_j) + i\sin(\phi_j)$  and its alias.<sup>7</sup> The generated cycle has periods of  $2\pi/\phi_j$  and periods of  $2\pi/(2\pi - \phi_j)$  for its alias.<sup>8</sup>

We use the recently developed non-parametric methodology of Bierens (2001) to test for the existence of complex unit roots by considering the distribution of the test statistic of Eq. (2) that reads:

$$\max \left\{ \frac{\rho(\phi_j)}{n}, \quad j = 1, \dots, k \right\} \tag{3}$$

Bierens shows that if  $y_t$  is a complex unit root process then the maximum of the distribution function must converge to a random variable that is bounded from below by

$$B(k) = \left( \sum_{m=1}^k \frac{\int_0^1 w_{1,m}(x)^2 dx + \int_0^1 w_{2,m}(x)^2 dx}{\left( \int_0^1 w_{1,m}(x) dx \right)^2 + \left( \int_0^1 w_{2,m}(x) dx \right)^2} \right), \tag{4}$$

<sup>7</sup> The alias is the corresponding cycle of the root  $\exp(j\phi_j)$  if  $(\phi_j) \in (0, \pi)$  and  $\exp(-j\phi_j)$  if  $(\phi_j) \in (\pi, 2\pi)$ .

<sup>8</sup> Diaz-Emparanza (2004) concludes that both cycles have the same cyclical behavior if the data generating process is discrete but it is impossible to distinguish them from their alias.

where  $(w_{1,m}, w_{2,m})$  is a set of independent Standard Brownian Motions (SBM). Under the null hypothesis that  $y_t$  is a complex unit root, it is shown that the  $\rho(\xi)/n$  has a razor-sharp at  $\xi = \phi$ . However, under the alternative stationary hypothesis,  $\rho(\xi)$  is bounded from above by

$$\rho(\xi) = \frac{|\eta e^{1\xi}|^2}{\sigma_y^2} (w_1, \varepsilon(1)^2, w_2, \varepsilon(1)^2) \tag{5}$$

which is an independent  $X^2$  random variable, times  $|\eta e^{1\xi}|^2 / \sigma_y^2$ .

Moreover, exchange rates stationary AR(p) null hypothesis can be tested against the alternative complex unit roots hypothesis by replacing the variance  $\sigma_\varepsilon^2$  and the lag polynomial  $\hat{\theta}_p$  of the AR(p) by their corresponding OLS estimates. The appropriate test statistic takes the form:

$$\hat{A}_{k,p} = \hat{\sigma}^{-2} \sum_{j=1}^k |\hat{\theta}_p(\exp(i\phi_j))|^2 \rho(\phi_j). \tag{6}$$

which is a set of independent  $\chi_2^2$  random variables under the stationary hypothesis. On the other hand, if some of the values of  $\phi_j$  correspond to complex-conjugate unit roots, then the tests statistic  $\hat{A}_{k,p}$  converges in distribution to infinity.

### 3. Empirical results

To begin the investigation for complex unit roots in the spot exchange rates, it is appealing to conduct the periodogram first to determine the potential complex unit root frequencies. The frequencies  $\phi_{0,1}, \dots, \phi_{0,k}$  correspond to  $k$  highest peaks are selected to avoid size distortions and lack of power. Tests of complex unit roots and formulation of the null hypothesis are set prior to looking at the data. No frequencies corresponding to cycle dates or durations stabilization have been set *a priori*.

Fig. 2 graphs the standardized periodogram of the transformed spot rates of the Canadian dollar, Swiss franc, and British pound, respectively. As shown in the figure, the first two peaks of the Canadian dollar periodogram (with dips in tops) keep up a correspondence to cycle durations of 204- and 91-month. The third peak corresponds to a cycle duration of 47-month. The last three highest peaks correspond to shorter cycle durations of 40-, 34-, and 27-month. For the Swiss franc, the figure indicates that the first and second peaks of the periodogram have cycle durations of 180- and 102-month. The third and highest peak corresponds to a cycle of 49-month. The last three peaks match the cycles of 35-, 24-, and 21-month. Finally, the figure indicates that a cycle of 103-month has the highest peak in the British pound periodogram. The next five highest peaks die out, respectively, and correspond to cycle durations of 73-, 49-, 34-, 27-, and 20-month.

We now turn to the main question posed by our paper—Is this apparent cyclicity of the exchange rates are related to complex unit roots? To answer this question, we test the null hypothesis that each spot rate has six pairs of complex-conjugate unit roots, with frequencies matching the highest cycles, as indicated earlier. As shown in Table 1, the hypothesis of six conjugate-complex unit roots cannot be rejected across all currencies. The  $B(K)$  statistics are highly insignificant with  $P$ -values around unity. The same results can be revealed if we test the null hypothesis that there is only one pair of complex unit roots. Cycles correspond to 224-, 47-, 40-, 34-, and 27-month for the Canadian dollar, 102-, 49-, 35-, 24-, and 21-month for the Swiss franc, and all of the cycles, in relation to the British pound are accepted to have a pair complex conjugate unit roots at 5% level.

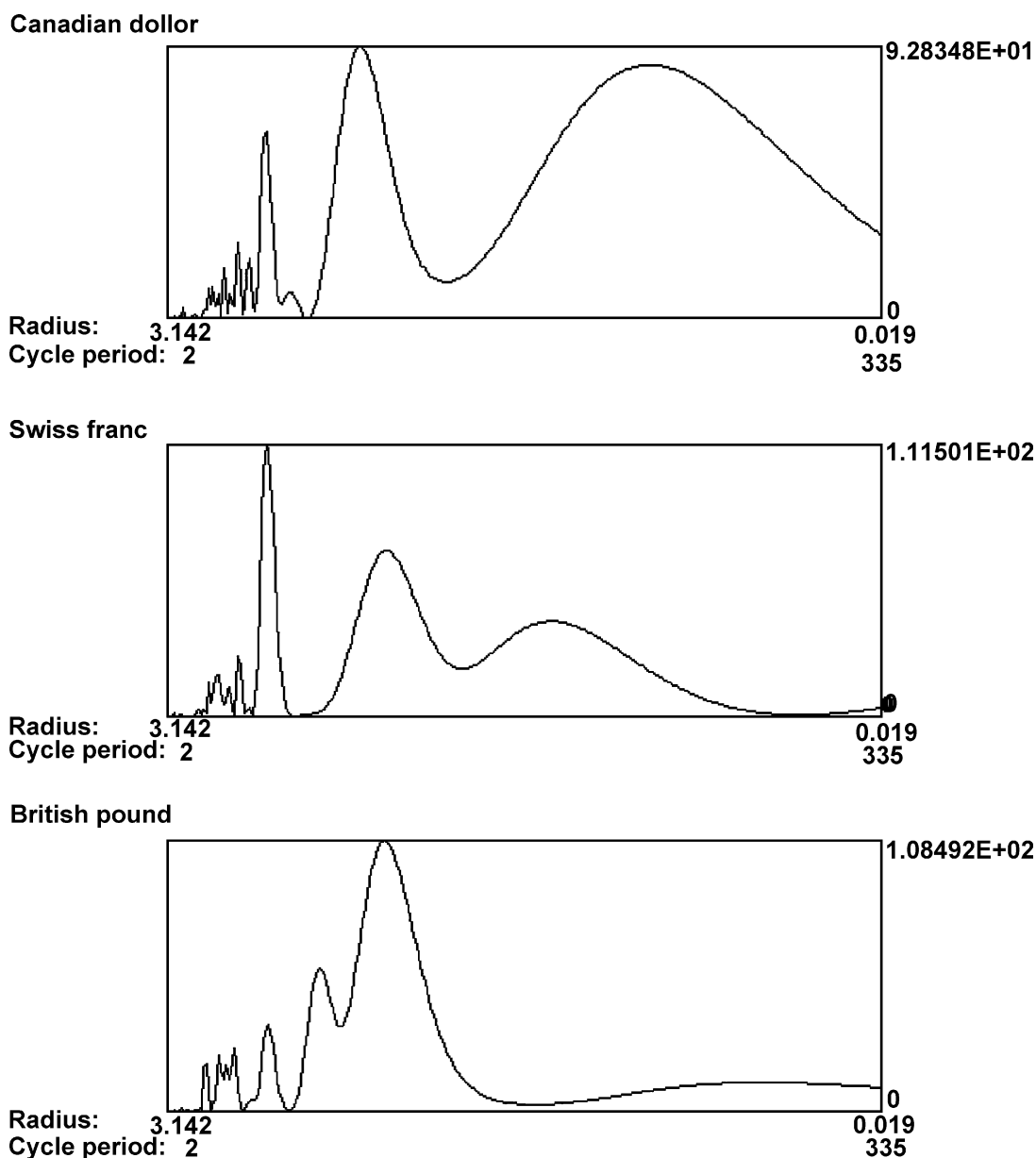


Fig. 2. Standardized continuous periodogram of the Canadian dollar, Swiss franc, and British pound spot exchange rates, respectively.

However, cycle durations of 224- and 91-month for the Canadian dollar, 180- and 102-month for the Swiss franc, and 103- and 73-month for the British pound are rejected to have complex unit roots at 10% significant level. Inspection of the cycle periods shows that all of the currencies share four out of six frequent depreciation (appreciation) cycles against the US dollar. Those cycles are accepted to have complex unit roots at 5% and 10% level, respectively. One tends to share [Feldstein \(1988\)](#) analysis of the exchange rates behavior: “the dollar has experienced three big swings”. The first of these is when all of the currencies were appreciating against the US dollar; between the beginning of 1977 and the end of 1979. Early in 1980, the US dollar began to climb until the end of 1984. This was followed by 3-year depreciation in the US dollar against those currencies. Thus, the apparent of long swigs in the exchange rates seems to be driven by complex conjugate unit roots that are persistent. The notable violations of this implication are cycles correspond to 91- and 180-month for the Canadian dollar and the Swiss franc, respectively

Table 1  
Null hypothesis and tests results of Bierens (2001) for spot exchange rates (1973–2002)

$J$	$\phi_{0,1}$	Cycle	$\hat{\rho}(\phi_{0,1})/n$
Canadian dollar			
1	0.02805	224	0.23454
2	0.06905	91	0.25137
3	0.13368	47	0.16894
4	0.15708	40	0.05638
5	0.18480	34	0.04961
6	0.22440	27	0.00269
Swiss franc			
1	0.03491	180	0.10645
2	0.06160	102	0.18285
3	0.12823	49	0.29730
4	0.17952	35	0.06741
5	0.26180	24	0.04249
6	0.29920	21	0.03942
British pound			
1	0.06100	103	0.29442
2	0.08607	73	0.15629
3	0.12823	49	0.09500
4	0.18480	34	0.05380
5	0.23271	27	0.04462
6	0.31416	20	0.05307

Notes. Test statistic =  $\max \hat{\rho}(\phi_{0,1})/n$  and  $P$ -values for the Canadian dollar, Swiss franc, and British pound are (0.2531, 0.9976), (0.2721, 0.9969) and (0.2973, 0.9962), respectively. 10% Critical region = (0.03314). 5% Critical region = (0.01988).

Next, we consider testing the stationary AR(p) hypothesis against the complex unit root alternative by using the  $\hat{A}_{k,p}$  test specified in Eq. (6). Findings from  $\hat{A}_{k,p}$  test are reported in Table 2. In order to examine the sensitivity of the results to the choice of lag length, we experimented with a wider set of lag lengths (1–48). According to Table 2,  $\hat{A}_{k,p}$  test points out that the autoregressive polynomials of the yearly changes in exchange rates are complex-unit-roots processes. The results are robust across all lag specifications. This confirms the above evidence that complex conjugate-unit roots drive the cyclical behavior of the exchange rates.

Table 2  
Test of the stationary AR(p) hypothesis on the basis of the periodogram ordinates

Canadian dollar				Swiss franc				British pound			
p	$\hat{A}_{k,p}$	p	$\hat{A}_{k,p}$	p	$\hat{A}_{k,p}$	p	$\hat{A}_{k,p}$	p	$\hat{A}_{k,p}$	p	$\hat{A}_{k,p}$
1	31.15	9	31.74	1	37.59	9	56.44	1	42.14	9	53.06
2	32.49	10	34.73	2	40.95	10	55.25	2	45.69	10	56.90
3	30.14	11	45.86	3	39.87	11	63.77	3	46.54	11	60.22
4	30.72	12	61.76	4	42.08	12	66.99	4	8.14	12	59.53
5	30.14	18	39.93	5	41.77	18	56.66	5	51.19	18	56.35
6	30.47	24	60.07	6	43.14	24	77.30	6	51.73	24	77.15
7	29.84	36	57.10	7	45.85	36	65.10	7	52.39	36	110.09
8	30.40	48	70.05	8	49.85	48	79.58	8	52.96	48	124.67

Notes. 10% critical region = (18.55), 5% critical region = (21.03).



#### 4. Conclusion

In this paper, we show that the long swings behavior of spot exchange rates is persistent and is due mainly to complex-conjugate unit roots in the autoregressive lag polynomial of the nominal spot exchange rates. In sharp contrast to the random walk theory of exchange rates, we find that the yearly changes in spot exchange rates have six complex-conjugate unit roots. The results are robust whether the hypothesis is set under the null or the alternative. The periodogram analysis reveals that the cycle durations of spot exchange rates are persistent and share the long swings view of Engel and Hamilton (1990).

#### References

- Abuaf, N., Jorion, P., 1990. Purchasing power parity in the long run. *Journal of Finance* 45, 157–174.
- Bazdresch, S., Werner, A., 2005. Regime switching models for the Mexican peso. *Journal of International Economics* 65, 185–201.
- Bergman, U.M., Hanson, J., 2005. Real exchange rates and switching regimes. *Journal of International Money and Finance* 24, 121–138.
- Bierens, H., 2001. Complex unit roots and business cycles: are they real? *Econometric Theory* 17, 962–983.
- Blough, S.R., 1992. The relationship between power and level for generic unit root tests in finite samples. *Journal of Applied Econometrics* 7, 295–308.
- Bollen, N.P.B., Gray, S.F., Robert, E.W., 2000. Regime switching in foreign exchange rates: evidence from currency option prices. *Journal of Econometrics* 94, 239–276.
- Chari, V.V., Kehoe, P., McGrattan, E.R., 1998. Monetary shocks and real exchange rates in sticky price models of international business cycles. NBER Working papers 5876.
- Choi, I., 2001. Unit root tests for panel data. *Journal of International Money and Finance* 20, 249–272.
- Cochrane, J.H., 1991. A critique of the application of unit root tests. *Journal of Economic Dynamics and Control* 15, 275–284.
- Dacco, R., Satchell, S., 1999. Why do regime-switching models forecast so badly? *Journal of Forecasting* 18, 1–16.
- Dewachter, H., 2001. Can Markov switching models replicate chartist profits in the foreign exchange market? *Journal of International Money and Finance* 20, 25–41.
- Diaz-Emparanza, I., 2004. A Note on the paper by H.J. Bierens: complex unit roots and business cycles: are they real? *Econometric Theory* 20, 636–637.
- Dornbusch, R., 1976. Expectations and exchange rate dynamics. *Journal of Political Economy* 84, 1161–1176.
- Engel, C., 1994. Can the Markov switching model forecast exchange rates? *Journal of International Economics* 36, 151–165.
- Engel, C., 2000. Long-run PPP may not hold after all. *Journal of International Economics* 57, 243–273.
- Engel, C., Hamilton, J.D., 1990. Long swings in the dollar: are they in the data and do markets know it? *American Economic Review* 80, 689–713.
- Faust, J., 1996. Near observational equivalence and theoretical size problems with unit root tests. *Econometric Theory* 12, 724–731.
- Feldstein, M., 1988. Let the market decide. *The Economist* 309, 21–24.
- Frankel, J., 1981. Flexible exchange rates, prices and the role of 'News': lessons from the 1970's. *Journal of Political Economy* 89, 665–705.
- Frankel, J.A., 1986. International capital mobility and crowding out in the U.S. economy: imperfect integration of financial markets or of goods markets? NBER Working papers 1773.
- Hakkio, C., 1986. Does the exchange rate follow a random walk? A Monte Carlo study of four tests for a random walk. *Journal of International Money and Finance* 5, 221–229.
- Huizinga, J., 1987. An empirical investigation of the long-run behavior of real exchange rates. *Carnegie-Rochester Conference Series on Public Policy* 27, 149–214.
- Kollmann, R., 1997. The exchange rate in a dynamic optimizing current account model with nominal rigidities: a quantitative investigation. IMF Working Paper.
- Levin, A., Lin, C.F., 1992. Unit-root test in panel data: asymptotic and finite sample properties. Discussion paper. University of California.

- Lothian, J.R., 1997. Multi-country evidence on the behavior of purchasing power parity under the current float. *Journal of International Money and Finance* 16, 19–35.
- Lucas, R., 1982. Interest rates and currency prices in a two-country world. *Journal of Monetary Economics* 10, 335–359.
- Mark, N.C., 1995. Exchange rates and fundamentals: evidence on long-horizon predictability. *American Economic Review* 85, 201–218.
- Meese, R., Rogoff, K., 1988. Was it real? The exchange rate interest differential relation over the modern floating rate period. *Journal of Finance* 43, 933–948.
- Obstfeld, M., Rogoff, K., 1995. Exchange rate dynamics redux. *Journal of Political Economy* 103, 624–660.
- Oh, K.Y., 1996. Purchasing power parity and unit root tests using panel data. *Journal of International Money and Finance* 15, 405–418.
- Papell, D., Theodoridis, H., 1998. Increasing evidence of purchasing power parity over the recent float. *Journal of International Money and Finance* 17, 41–50.
- Rogoff, K.S., 2002. Why are G-3 exchange rates so fickle? *Finance & Development* 39, No. 2 (June).
- Roll, R., 1979. Violations of purchasing power parity and their implications for efficient international commodity markets. In: *International Finance and Trade*, vol. 1. Massachusetts, Cambridge.
- Stockman, A.C., 1987. The equilibrium approach to exchange rates. *Federal Reserve Bank of Richmond Economic Review* 73, 12–31.
- Stockman, A.C., 1990. International transmission and real business cycle models. *American Economic Review* 80, 134–138.
- Stockman, A.C., Tesar, L., 1995. Tastes and technology in a two-country model of the business cycle: explaining international comovements. *American Economic Review* 85, 168–185.
- Svensson, L.E., 1985. Currency prices, terms of trade, and interest rates: a general equilibrium asset-pricing, cash-in-advance approach. *Journal of International Economics* 18, 17–41.
- Taylor, A.M., 2002. A century of purchasing power parity. *The Review of Economics and Statistics* 84, 139–150.
- Williamson, J., Milner, C.R., 1991. *The World Economy: A Text Book in International Economics*. Harvester-Wheatsheaf.