# EE 207

# Foundation of Electrical and Computer Engineering

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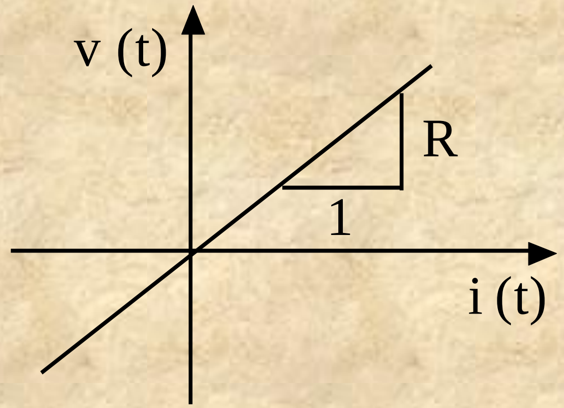
# Chapter 2

# Resistive Circuits

## Ohm's law :

The voltage across a resistor is directly proportional to the current flowing through it.




**V (t) = R i(t)** **$R \geq 0$**

The symbol of ohm is ($\Omega$)

$$1\Omega = \frac{1V}{1A}$$

The instantaneous power P (t):

$$P(t) = v(t)i(t) = R\,i(t)i(t)$$

$$= R\,i^2(t) = v(t)\frac{v(t)}{R} = \frac{v^2(t)}{R}$$

$$\therefore\ P(t) = v(t)i(t) = R\,i^2(t) = \frac{v^2(t)}{R}$$

***Note:*** Last equation says that the power at a resistor is always positive

$\Rightarrow$ Resistors always absorb power.

The inverse of resistance is <u>conductance</u>

$$G = \frac{1}{R}$$

The unit of conductance is Siemens (S)

$$1S = \frac{1A}{1V}$$

The current can be also expressed as

$$i(t) = G\,V(t)$$

And the instantaneous power is

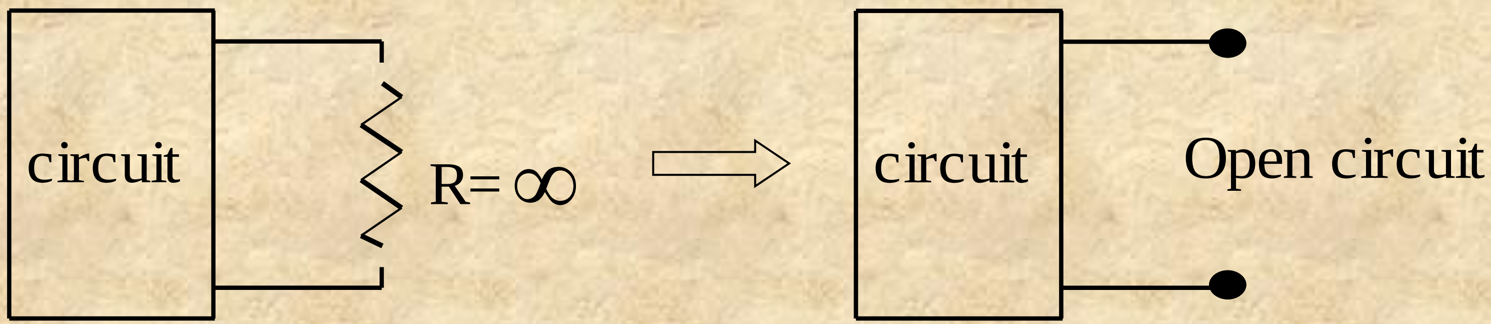
$$P(t) = v(t)i(t) = \frac{i(t)}{G}i(t) = \frac{i^2(t)}{G}$$

$$P(t) = v(t)i(t) = v(t)G\,v(t) = G\,v^2(t)$$

$$\Rightarrow\ = v(t)i(t) = \frac{i^2(t)}{G} = G\,v^2(t)$$

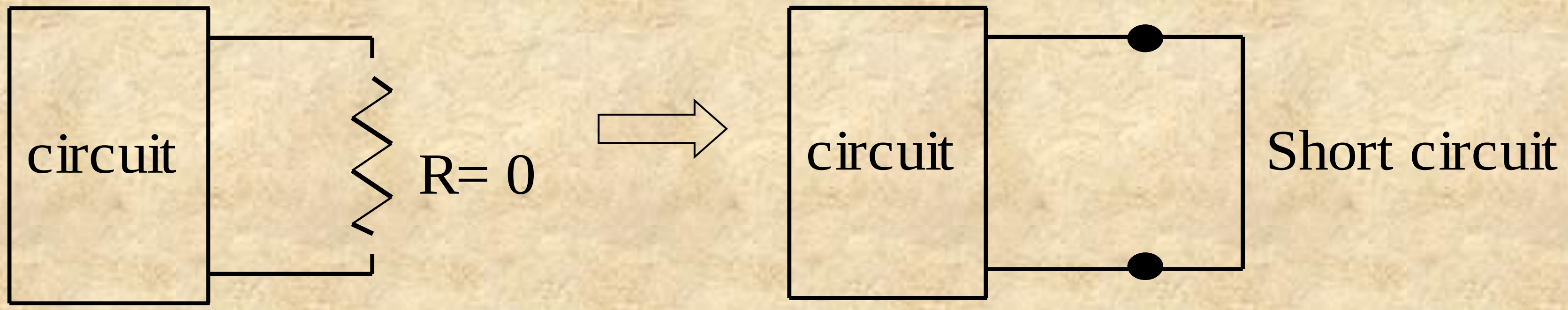
## <u>**Open and short Circuits**</u>

<u>Open circuit</u> $(R = \infty) \Rightarrow G = 0$




$$i(t) = \frac{v(t)}{R} = \frac{v(t)}{\infty} = 0$$

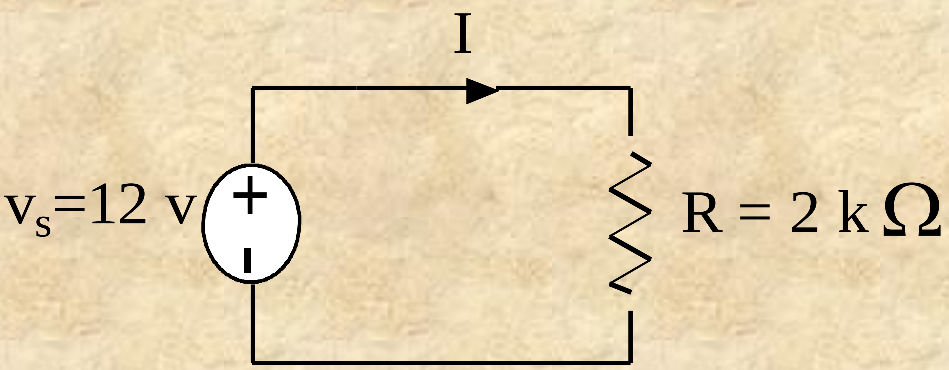
Short circuit ( R = 0) ⇨ $G = \infty$




$$v(t) = Ri(t) = 0 * i(t) = 0$$

# Example :

Consider the circuit:



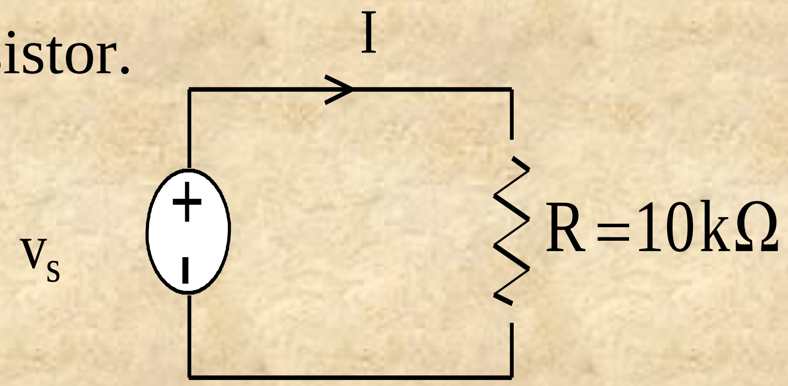

Find the current and power absorbed by the resistor

$$I = \frac{v_s}{R} = \frac{12\,v}{2\,k\Omega} = 6\,mA$$

$$P = v_R\,I = (12)(6m) = 72\,mw$$

**Example**:

The power absorbed by a 10 kΩ resistor in the circuit is 3.6 mW. Find voltage and current in the resistor.




$$P = V_s I = I^2 R$$

$$I^2 = \frac{P}{R}$$

$$I = \sqrt{\frac{P}{R}} = \sqrt{(3.6*10^{-3})/(10*10^{3})}$$

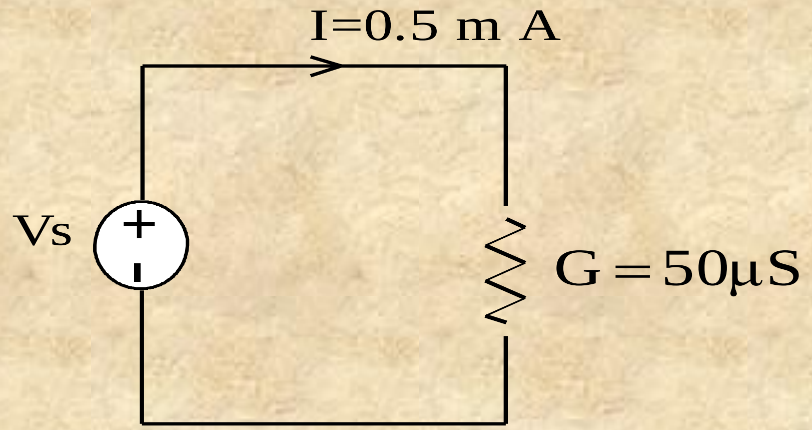
$$I = \sqrt{3.6*10^{-7}} = 0.6\,\text{mA}$$

$$V = IR = (0.6\,\text{mA})(10\,\text{k}\Omega)$$

$$V = 6\,\text{V}$$

# Example :

Find the value of the voltage source and the power absorbed by the resistance



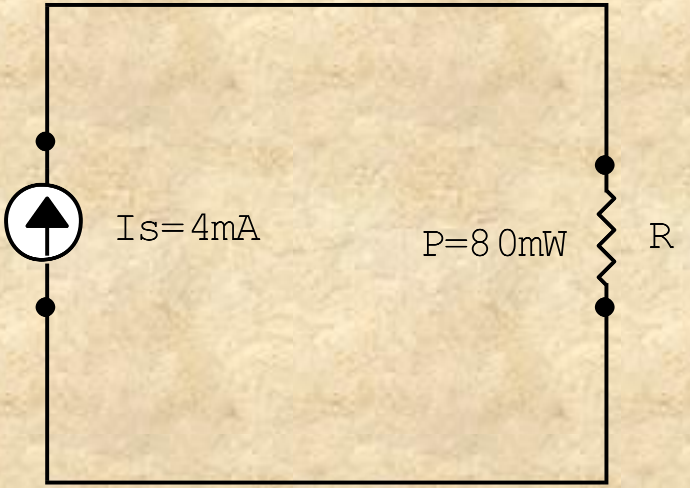

$$G = 50\mu S \Longrightarrow R = 1/G = 2*10^4$$

$$V_s = IR = (0.5mA)(20*10^4\Omega) = 10V$$

$$P_R = IV = (10V)(0.5mA) = 5mW$$

## Example :

Find R and the voltage across
The resistor?




$$P = V\,Is$$

$$V = \frac{P}{Is} = \frac{80*10^{-3}\ W}{4*10^{-3}\ A}$$

$$V = 20V.$$

$$V = I\,R = (4*10^{-3}\ A)R$$

$$R = \frac{V}{I} = \frac{20\ V.}{4*10^{-3}\ A} = 5k\Omega$$

# Kirchoff's Laws:

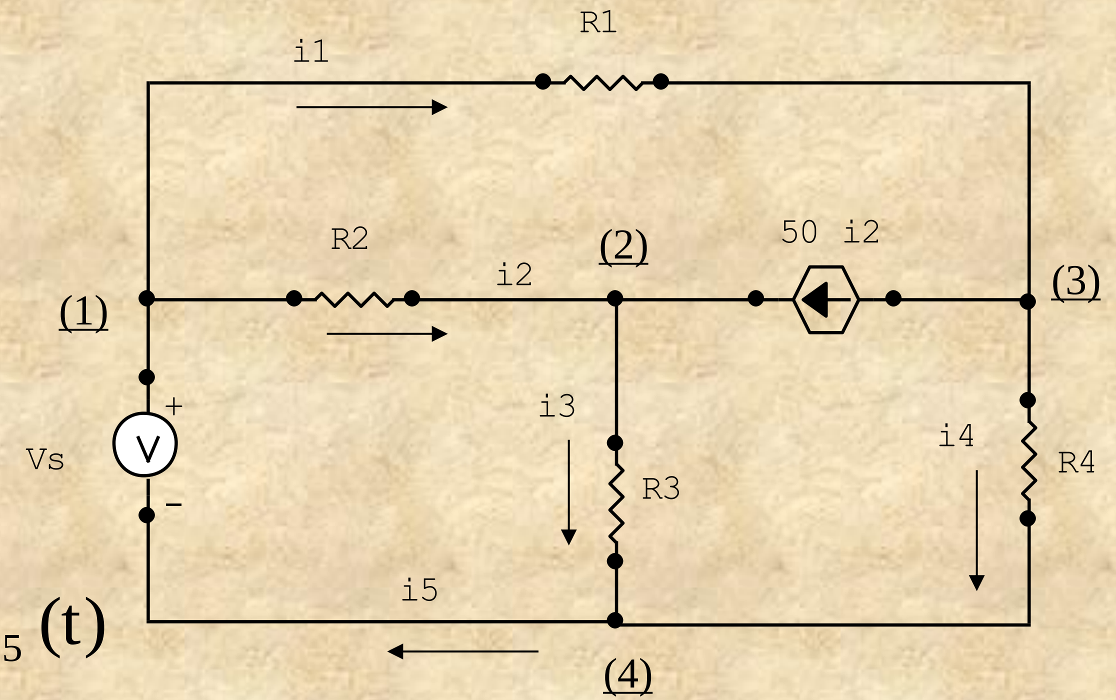
**(1) kirchoff's current law (KCL) :**
the sum of all currents entering any node is zero.

$$\Longrightarrow \qquad \sum_{k=1}^{N} i_k(t) = 0$$

Where N= number of currents.

**Example:**

Write the KCL equation




Here we have (4) nodes:

At node (1) : $i_1(t)+i_2(t)=i_5(t)$

At node (2) : $i_2(t)+50i_2(t)=i_3(t)$

At node (3) : $50i_2(t)+i_4(t)=i_1(t)$

At node (4) : $i_3(t)+i_4(t)=i_5(t)$

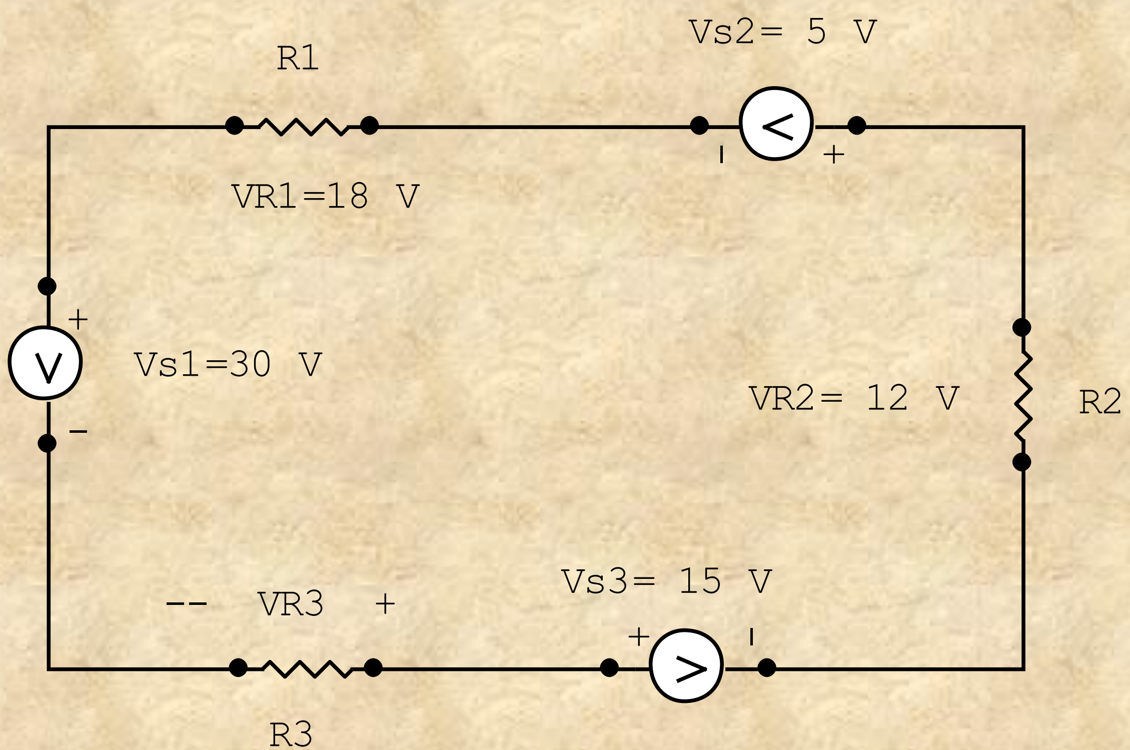
## (2) Kirchoff's voltage Law (KVL):

The sum of the voltage around any loop is zero.

$$\sum_{k=1}^{N} v_k(t) = 0 \quad \Rightarrow \quad N = \text{\# of voltage}$$

Example:

Find $V_{R3}$ ? using KVL




$-30+18-5+12-15+V_{R3} = 0$

$V_{R3} = 20\ V$

# Example :

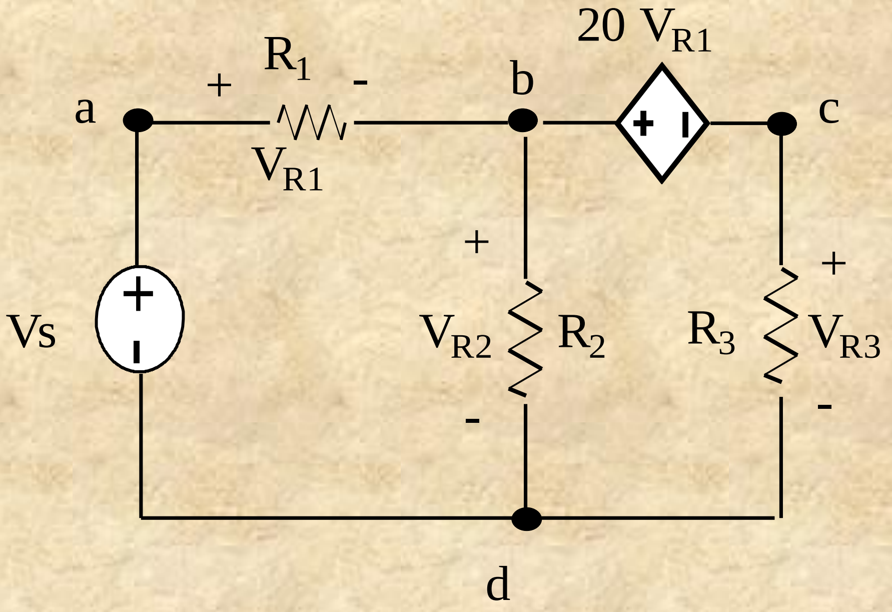
Find the KVL equation for the two paths abda and bcdb

## Path abda:

$$v_{R1} + v_{R2} - v_s = 0$$

## Path bcdb:

$$20 v_{R1} + v_{R3} - v_{R2} = 0$$



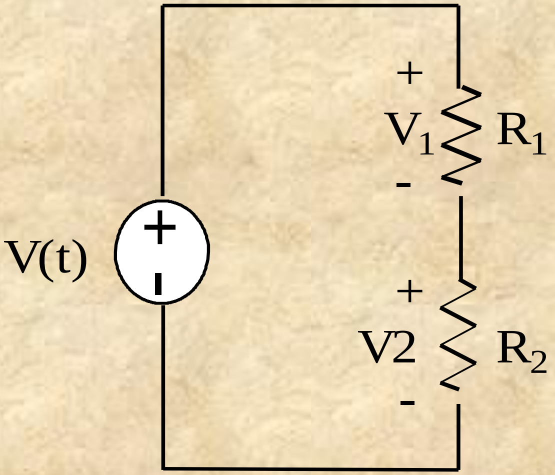
# Single Loop circuits

We will discuss (2) issues :

1. **Voltage divider rule:**
   Voltage is divided between resistor in direct proportion to their resistance

$$v_1(t) = \frac{R_1}{R_1 + R_2} v(t)$$

$$v_2(t) = \frac{R_2}{R_1 + R_2} v(t)$$



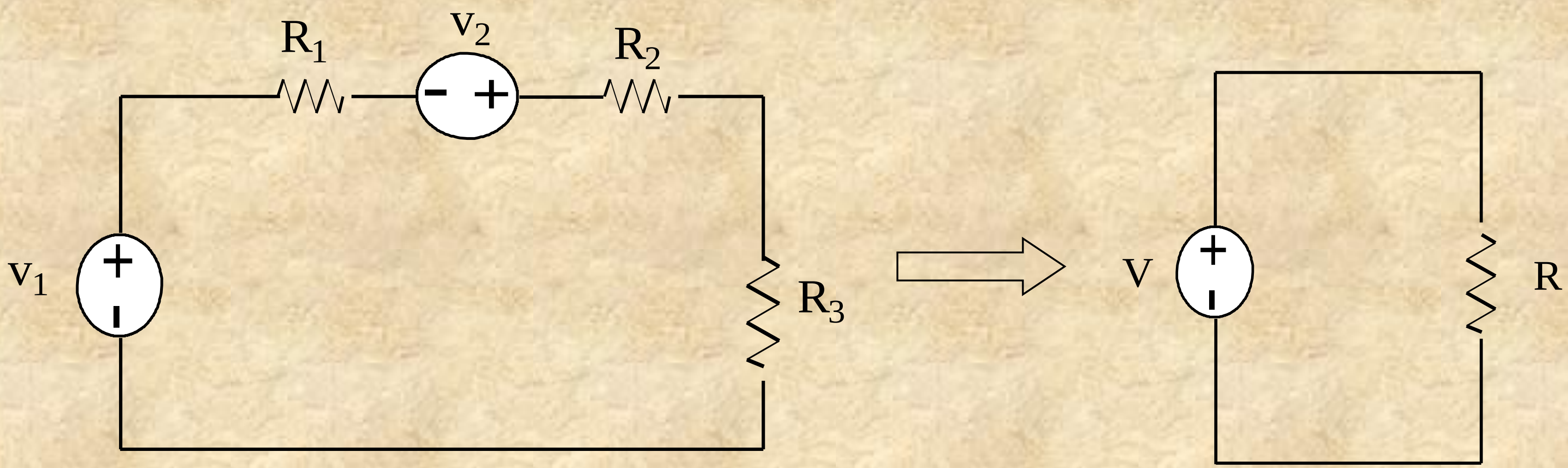

How?

$$v_1 = R_1 i = R_1 \left(\frac{v}{R_1 + R_2}\right)$$

$$v_1 = \frac{R_1}{R_1 + R_2} v$$

## Multi Sources / resistors :

- Source can be added $v = v_1 + v_2 + \ldots\ldots$
- Resistors can be added $R = R_1 + R_2 + \ldots..$



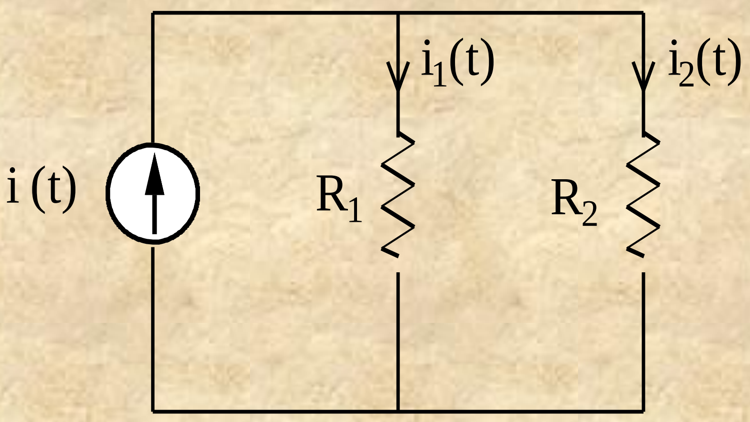

Where:

$v = v_1 + v_2$

$R = R_1 + R_2 + R_3$

# Single Node-Pair circuits :

We will discuss (2) issues:




1. **Current-divider Rule .**

$$i_1(t) = \frac{R_2}{R_1 + R_2} i(t)$$

$$i_2(t) = \frac{R_1}{R_1 + R_2} i(t)$$

**Why ??**

$$v = i_1 R_1 = i_2 R_2$$

$$\therefore i_1 = \frac{R_2}{R_1} i_2$$

$$i = i_1 + i_2 \quad \Rightarrow \quad i_2 = i - i_1$$

$$i_1 = \frac{R_2}{R_1}(i - i_1)$$

$$i_1 (1 + \frac{R_2}{R_1}) = \frac{R_2}{R_1} i$$

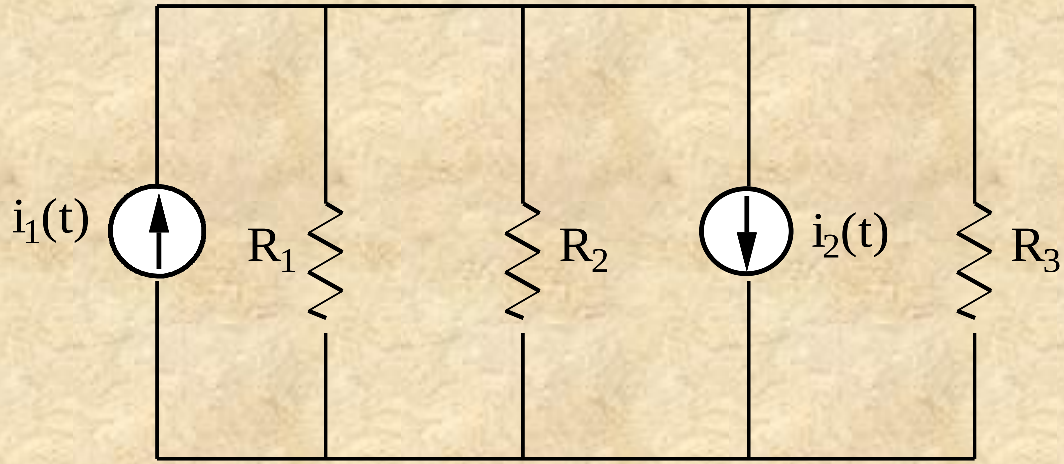
$$i_1 (\frac{R_1 + R_2}{R_1}) = \frac{R_2}{R_1} i$$

$$i_1 = \frac{R_2}{R_1 + R_2} i$$

**2. Multiple sources/resistors** :

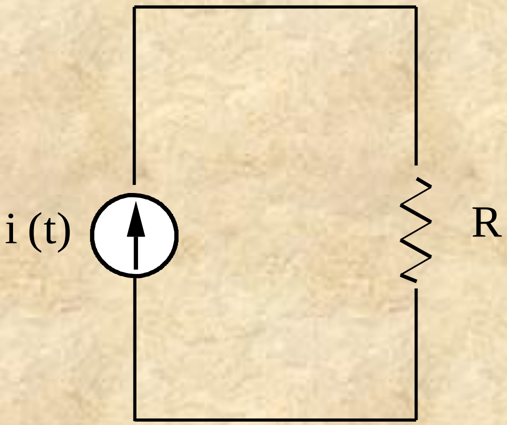
•Current source can be added.

•Resistors can added as reciprocals




$$i(t) = i_1(t) + i_2(t)$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



# Series and parallel resistors :

Series :

$$R = R_1 + R_2 + \ldots + R_N \quad \Rightarrow R_s = \sum_{k=1}^{N} R_k$$

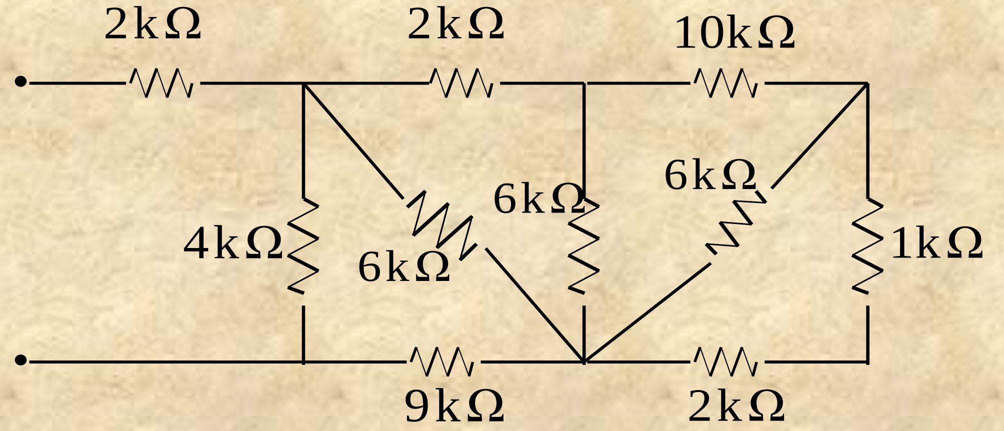
Parallel

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$$

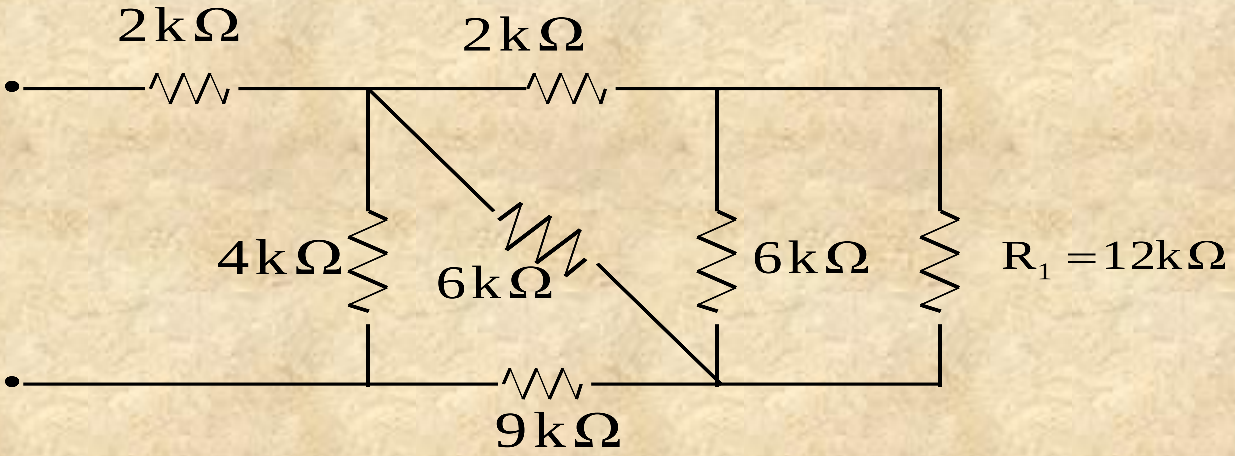
$$\frac{1}{R_P} = \sum_{k=1}^{N} \frac{1}{R_k}$$

**Example :**

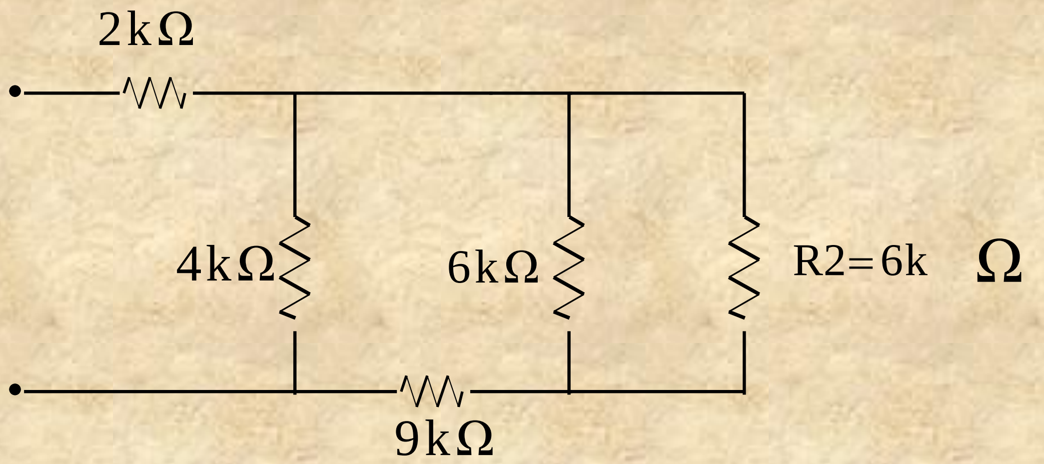
Find equivalent resistance



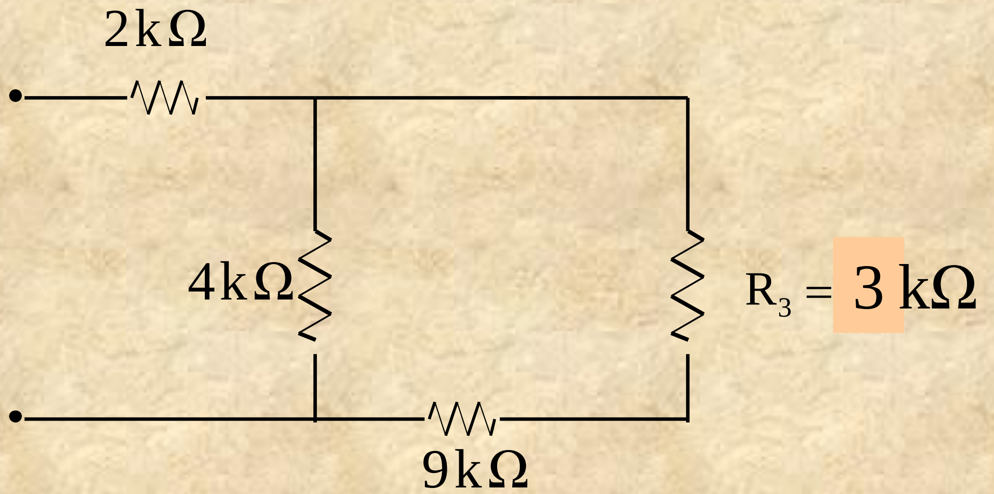

$$R_1 = \{[(1k\Omega) + (2k\Omega)] \| 6k\Omega\} + 10k\Omega$$



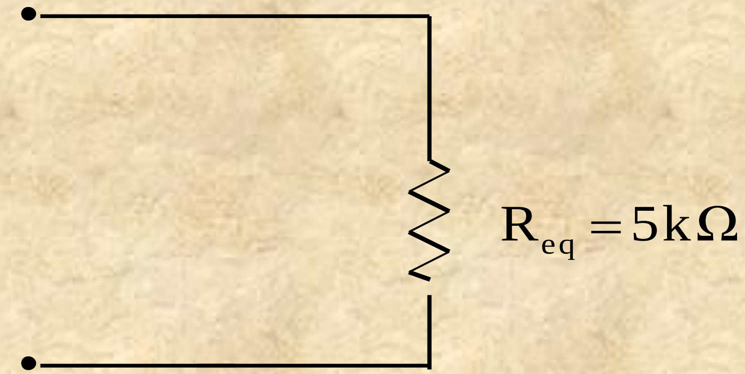
$$R2 = [12k \,//\, 6k] + 2k = 6k$$




$$R_3 = (6k \,//\, 6k) = 3k \quad \Omega$$

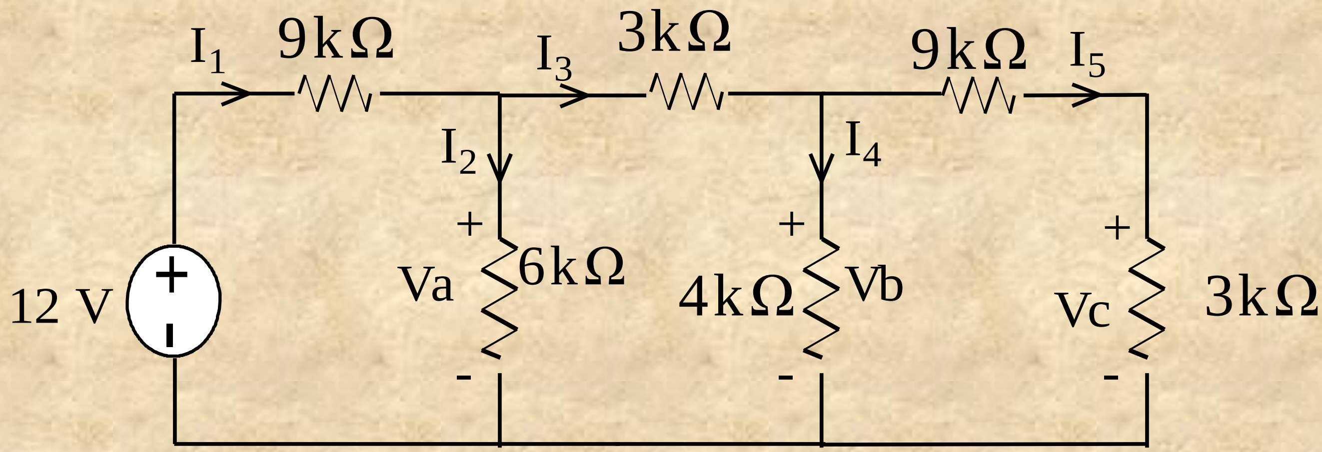



$$R_{eq} = (12k \parallel 4k) + 2k = 5k$$

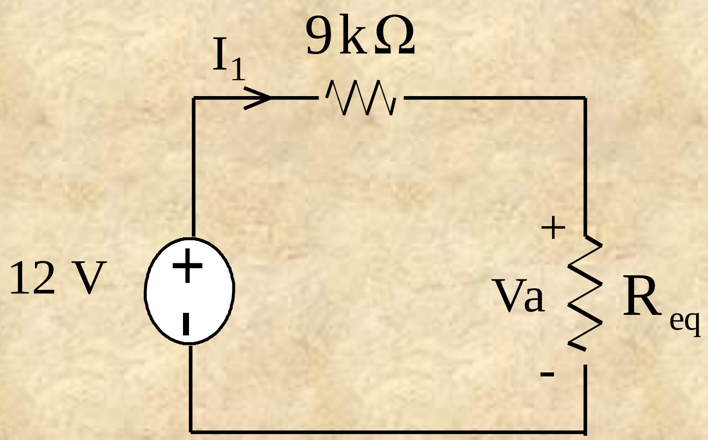


**Example :**

Find all currents and voltages




The equivalent circuit is :



$$R_{eq} = \left[\left[(3k + 9k) // 4k\right] + 3k\right] // 6k$$

$$= 3k\Omega$$

$$I_1 = \frac{12V}{9k+3k} = 1mA \qquad \Rightarrow V_a = R_{eq} I_i = 3V$$

$$\therefore \quad I_2 = \frac{V_a}{6k\Omega} = \frac{3}{6k} = \frac{1}{2} mA$$

$$I_3 = I_1 - I_2 \qquad \Rightarrow I_3 = 1mA - \frac{1}{2} mA = \frac{1}{2} mA$$

$$V_{3k\Omega} = I_3(3k\Omega) = (\frac{1}{2}mA)(3k\Omega) = 1.5V$$

$$\therefore \quad V_b - V_a + V_{3k\Omega} = 0$$

$$V_b - 3 + 1.5 = 0 \qquad \Rightarrow V_b = 1.5V$$

$$I_5 = \frac{V_b}{9k + 3k} = \frac{1.5}{12k} = \frac{1}{8}mA$$

$$V_c = I_5(3k\Omega) = \frac{1}{8}mA(3k\Omega) = \frac{3}{8}V$$

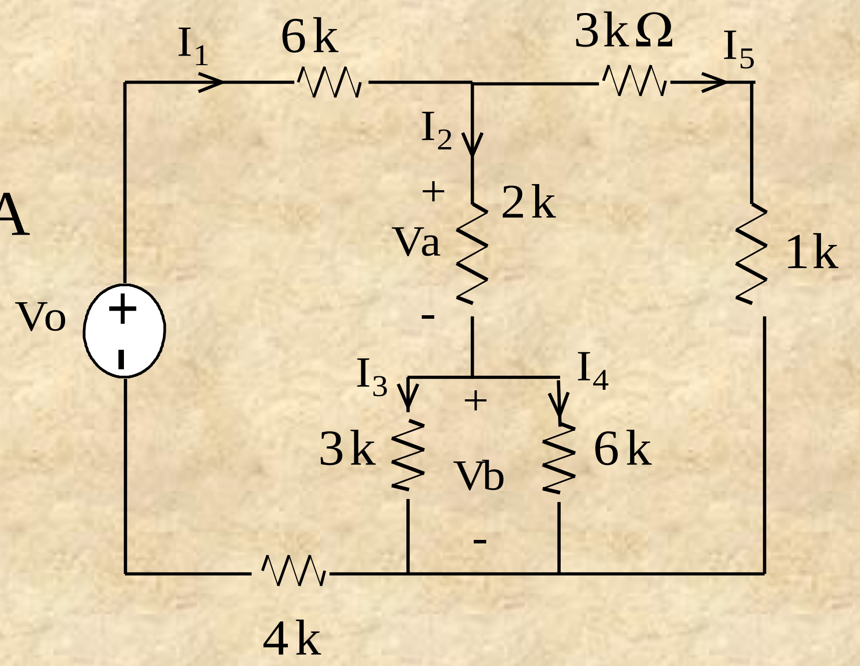
**Example :**

Find the source voltage Vo if $I_4$=1/2 m A ?

$$V_b = I_4\,(6k) = (\frac{1}{2}\,mA)(6\,k\Omega) = 3V$$

$$\therefore \quad I_3 = \frac{V_b}{3k} = \frac{3}{3k} = 1mA$$

$$I_2 = I_3 + I_4 = 1mA + \frac{1}{2}mA = 1.5mA$$



$$\therefore \quad V_a = 2k\Omega\, I_2 = (2k)(1.5m) = 3V$$

$$I_5 = \frac{V_a + V_b}{3k + 1k} = \frac{3+3}{4k} = 1.5mA$$

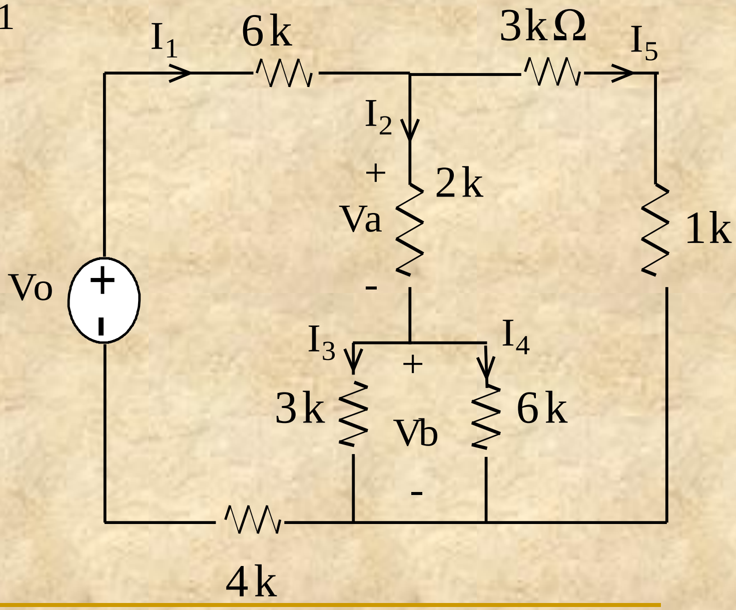
$$\therefore \quad I_1 = I_2 + I_5 = 1.5mA + 1.5mA = 3mA$$

$$V_0 = (6k\Omega)\, I_1 + V_a + V_b + (3k\Omega + 1k\Omega) I_1$$

$$V_0 = 6k(3m) + 3 + 3 + 4k(3m)$$

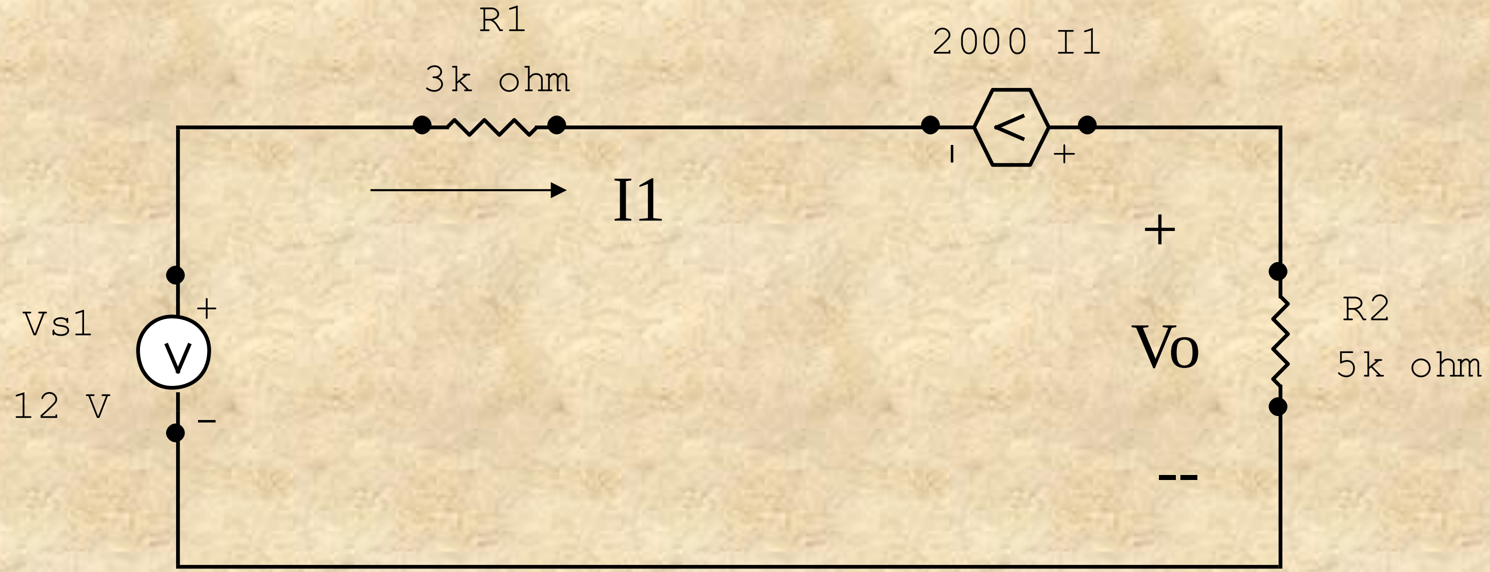
$$V_0 = 18 + 6 + 12 = 24$$

$$V_0 = 36V$$



**Example :**

Find $V_0$ ? Using KVL




$$-12 + 3k(I_1) - 2000 I_1 + 5k(I_1) = 0$$

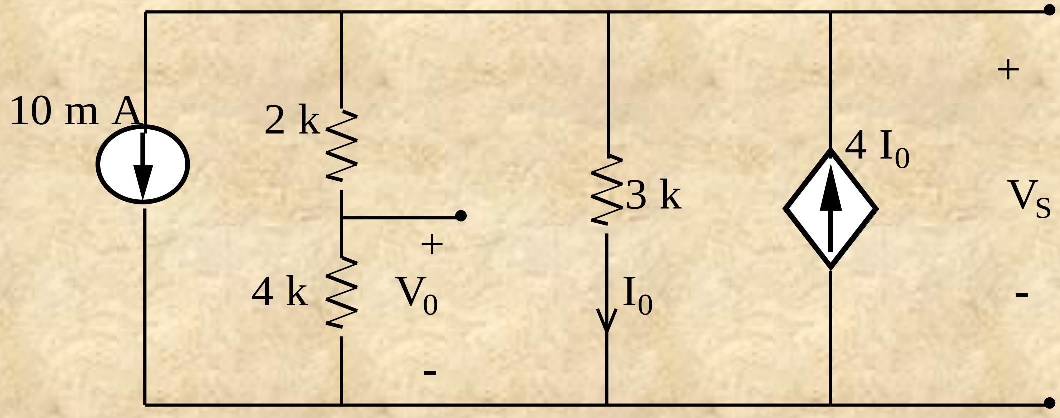
$$6kI_1 = 12 \quad \Rightarrow I_1 = 2\,mA$$

$$V_0 = (5k\Omega)(I_1) = (5k)(2m)$$

$$V_0 = 10\,V$$

**Example:**

Find $V_0$ using KCL:




$$10m + \frac{V_s}{6k} + \frac{V_s}{3k} - 4\left(\frac{V_s}{3k}\right) = 0$$

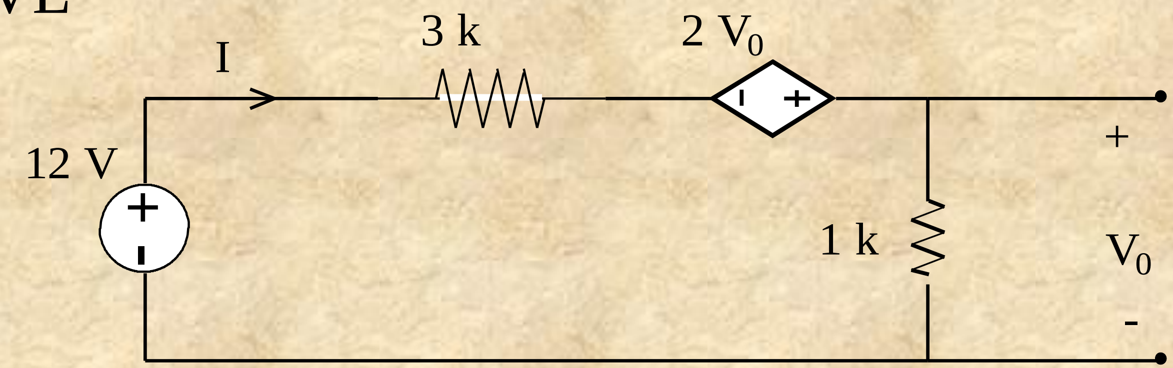
$$V_s\left[\frac{1}{6k} + \frac{1}{3k} - \frac{4}{3k}\right] = -10\,mA$$

$$V_s = 12\,V$$

$$V_0 = \frac{4k}{2k+4k}V_s = \frac{2}{3}(12) = 8\,V$$

**Example:**

Find $V_0$ using KVL




$$-12+(3k)I-2V_0+V_0=0$$

$$-V_0+(3k)I-12=0$$

$$I=\frac{V_0}{1k}$$

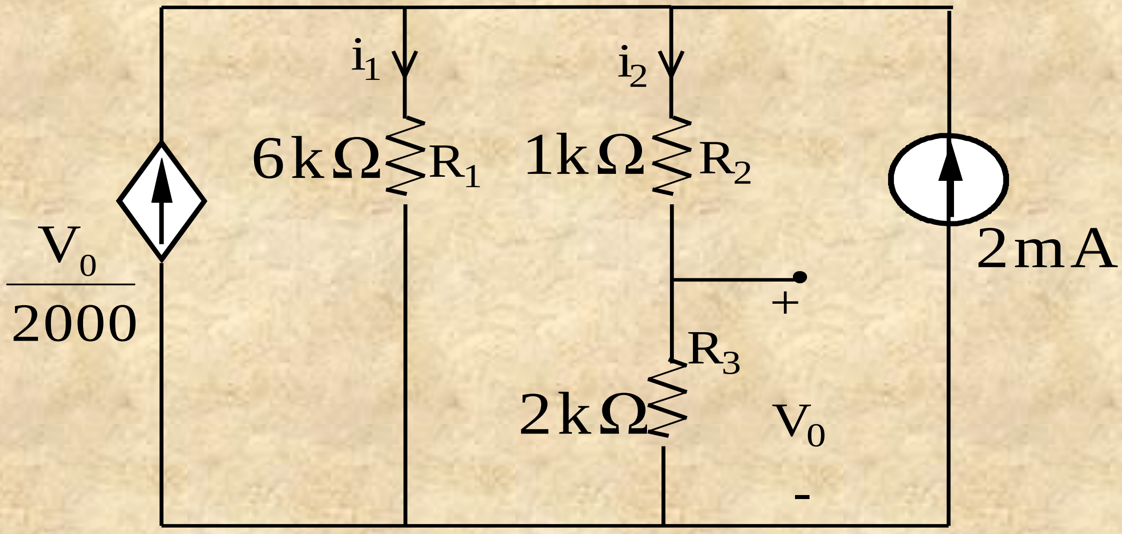
$$-V_0+(3k)\left(\frac{V_0}{1k}\right)=12$$

$$-V_0+3V_0=12$$

$$V_0=6V$$

**Example:**

Find $V_0$ in the network




$$\frac{V_0}{2000} - i_1 - i_2 + 2\,mA = 0$$

$$R_1 i_1 = (R_2 + R_3) i_2$$

Also $(6\,k\,\Omega) i_1 = (3\,k\,\Omega)\, i_2$

$$\therefore \quad i_1 = \frac{1}{2} i_2$$

$$\therefore \quad \frac{V_0}{2000} - \frac{1}{2} i_2 - i_2 + 2\,mA = 0$$

$$\frac{V_0}{2000} - \frac{3}{2} i_2 + 2\,mA = 0$$

$$\because \quad V_0 = R_3 i_2 \quad \Rightarrow \quad i_2 = \frac{V_0}{2k}$$

$$\therefore \quad \frac{V_0}{2k} - \frac{3}{2}\left(\frac{V_0}{2k}\right) + 2\,mA = 0$$

$$\frac{1}{2}\left(\frac{V_0}{2k}\right) = 2\,mA \quad \Rightarrow \quad V_0 = 8\,V$$