

# Chapter (4)

## Additional Analysis Techniques

Here we will study four additional Techniques

- Superposition
- Source transformation
- Thevenin and Norton Theorems
- Maximum power principle

### 1. Superposition :

#### Definition :

Whenever a linear circuit is excited by more than one independent source, the total response is the algebraic sum of individual responses

The idea is to activate one independent source at a time to get individual response.

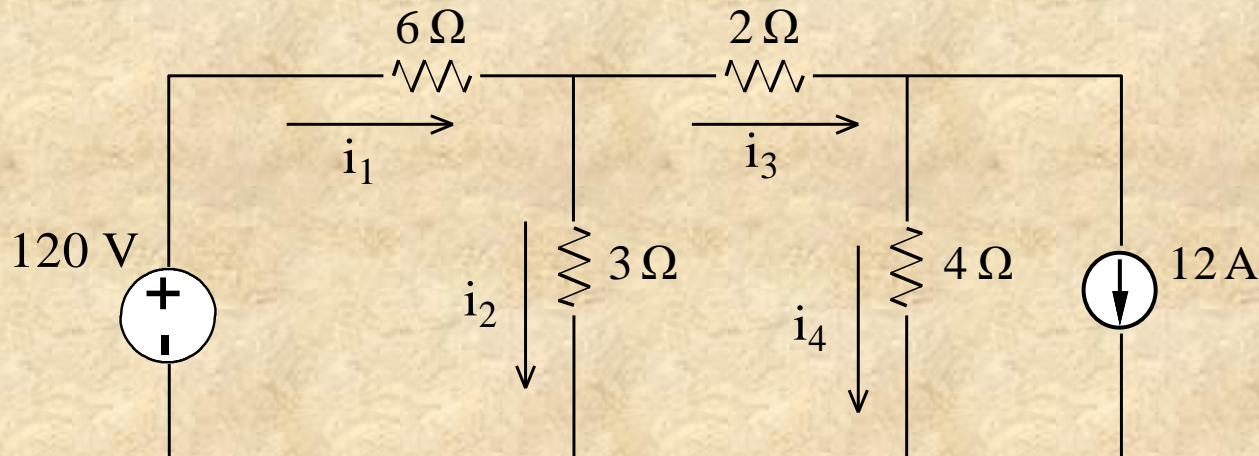
Then add the individual response to get total response

## Note:

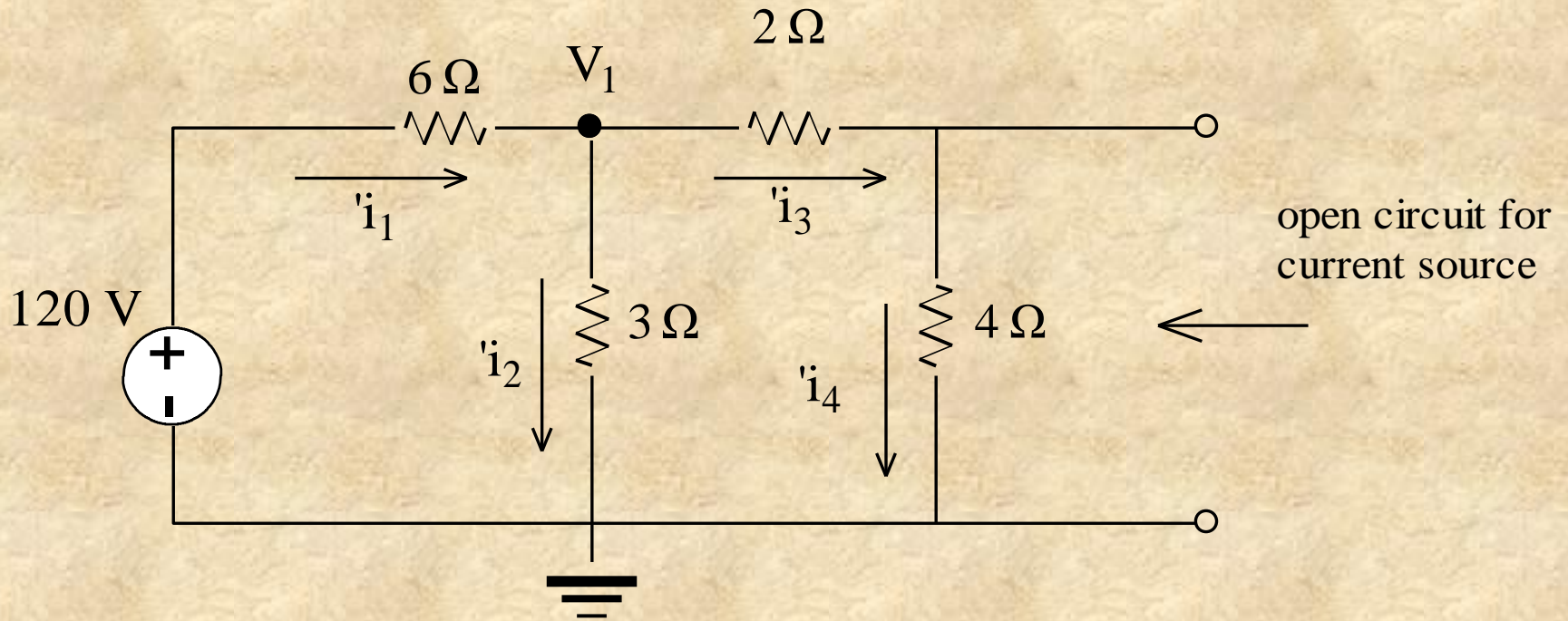
1. Dependent source are Never deactivated (always active)
2. When an independent voltage source is deactivated, it is set to zero.  
⇒ replaced by short circuit
3. When an independent current source is deactivated, it is set to zero.  
⇒ replaced by open circuit

## Example:

Use superposition to find  $i_1, i_2, i_3, i_4$  ?



- Activate independent voltage source 120 V only



- Using KCL at  $V_1$  (nodal analysis)

$$i_1 - i_2 - i_3 = 0$$

$$\frac{120 - V_1}{6} - \frac{V_1}{3} - \frac{V_1}{2+4} = 0$$

$$20 - V_1 \left( \frac{1}{6} + \frac{1}{3} + \frac{1}{6} \right) = 0$$

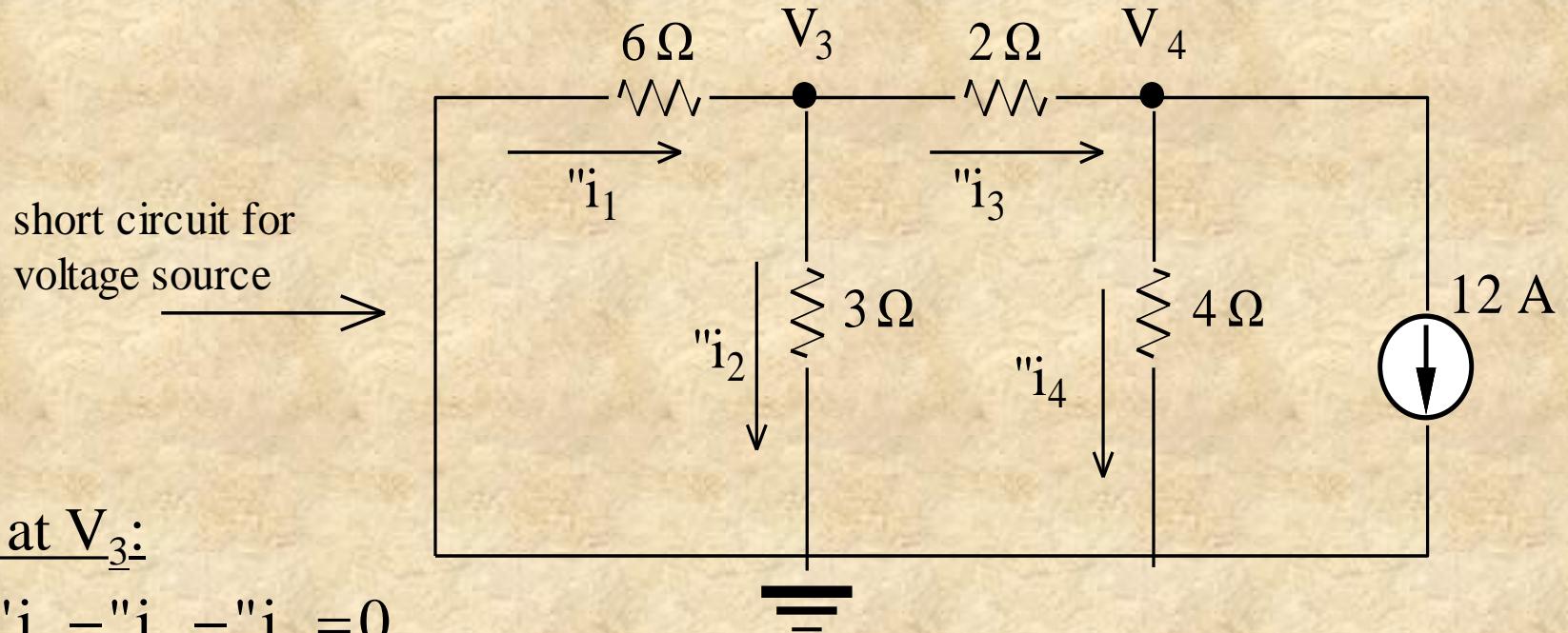
$$\Rightarrow V_1 = 30 \text{ V}$$

$$i_1 = \frac{120 - V_1}{6} = \frac{90}{6} = 15 \text{ A}$$

$$i_2 = \frac{V_1}{3} = \frac{30}{3} = 10 \text{ A}$$

$$i_3 = i_4 = \frac{V_1}{6} = 5 \text{ A}$$

\* Activate the independent current source only



KCL at V<sub>3</sub>:

$$i_1 - i_2 - i_3 = 0$$

$$\frac{-V_3}{6} - \frac{V_3}{3} - \frac{V_3 - V_4}{2} = 0$$

$$-V_3 - 2V_3 - 3(V_3 - V_4) = 0$$

$$-6V_3 + 3V_4 = 0 \quad \dots\dots(1)$$



KCL at V4:  $i_3 - i_4 - 12 = 0$

$$\frac{V_3 - V_4}{2} - \frac{V_4}{4} - 12 = 0$$

$$2 V_3 - 2 V_4 - V_4 = 48$$

$$2 V_3 - 3 V_4 = 48 \quad \dots\dots(2)$$

$$V_3 = -12 \text{ V}$$

$$V_4 = -24 \text{ V}$$

$$"i_1 = \frac{-V_3}{6} = \frac{12}{6} = 2 \text{ A}$$

$$"i_2 = \frac{V_3}{3} = \frac{-12}{3} = -4 \text{ A}$$

$$"i_3 = \frac{V_3 - V_4}{2} = \frac{-12 + 24}{2} = 6 \text{ A}$$

$$"i_4 = \frac{V_4}{4} = \frac{-24}{4} = -6 \text{ A}$$

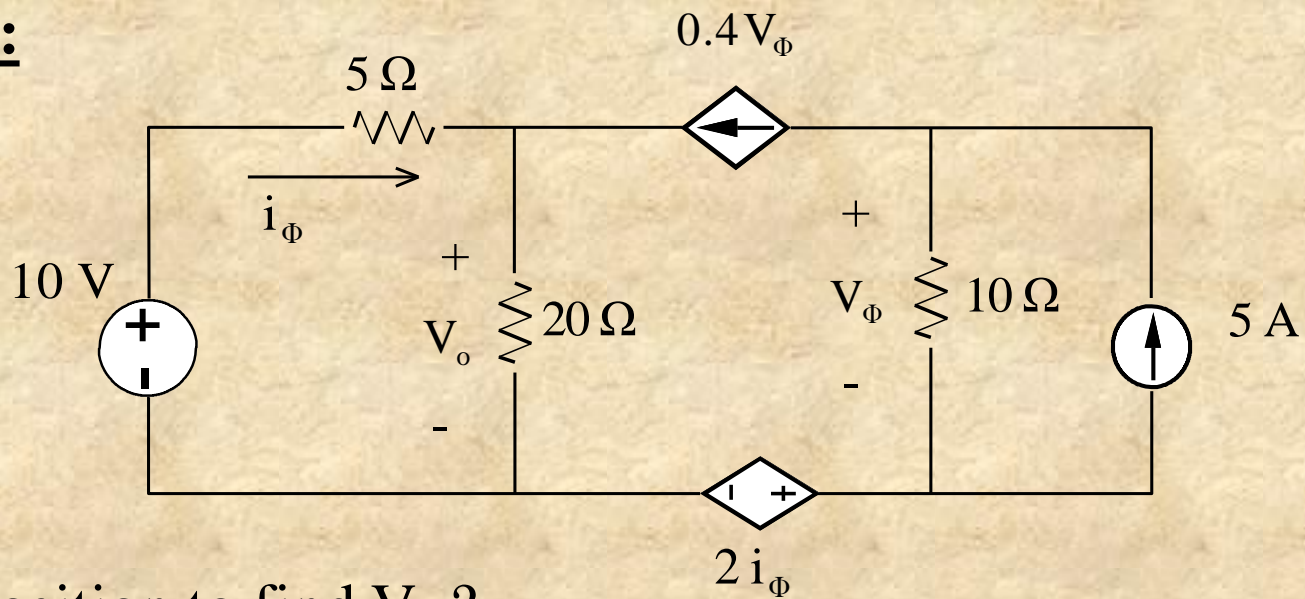
$$i_1 = 'i_1 + "i_1 = 15 + 2 = 17 \text{ A}$$

$$i_2 = 'i_2 + "i_2 = 10 - 4 = 6 \text{ A}$$

$$i_3 = 'i_3 + "i_3 = 5 + 6 = 11 \text{ A}$$

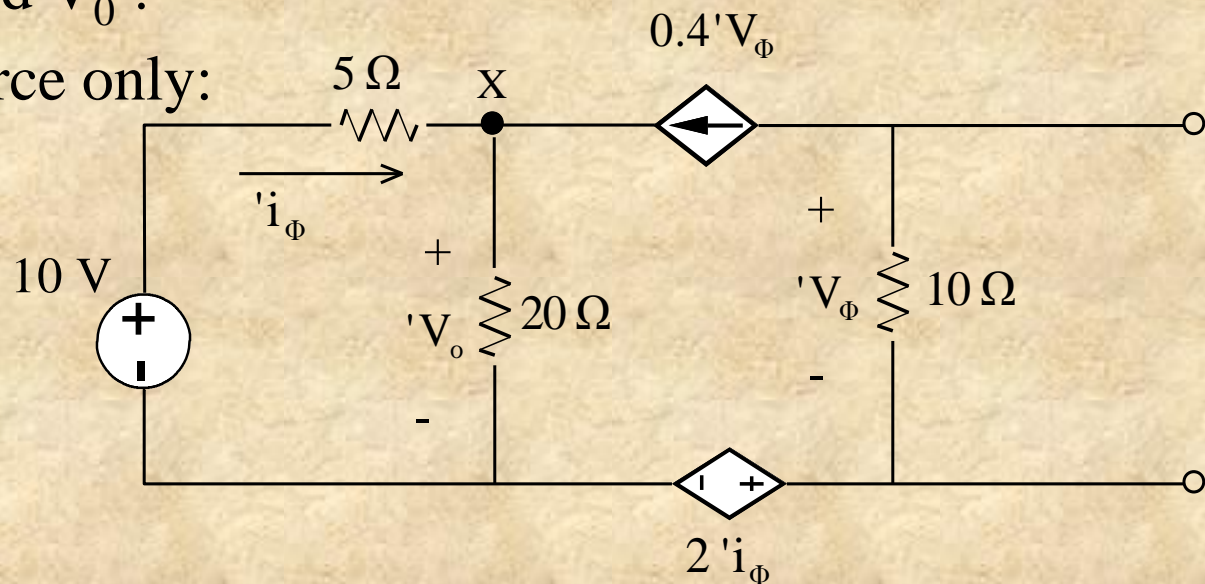
$$i_4 = 'i_4 + "i_4 = 5 - 6 = -1 \text{ A}$$

## Example :



Use super position to find  $V_o$  ?

⇒ Activate voltage source only:



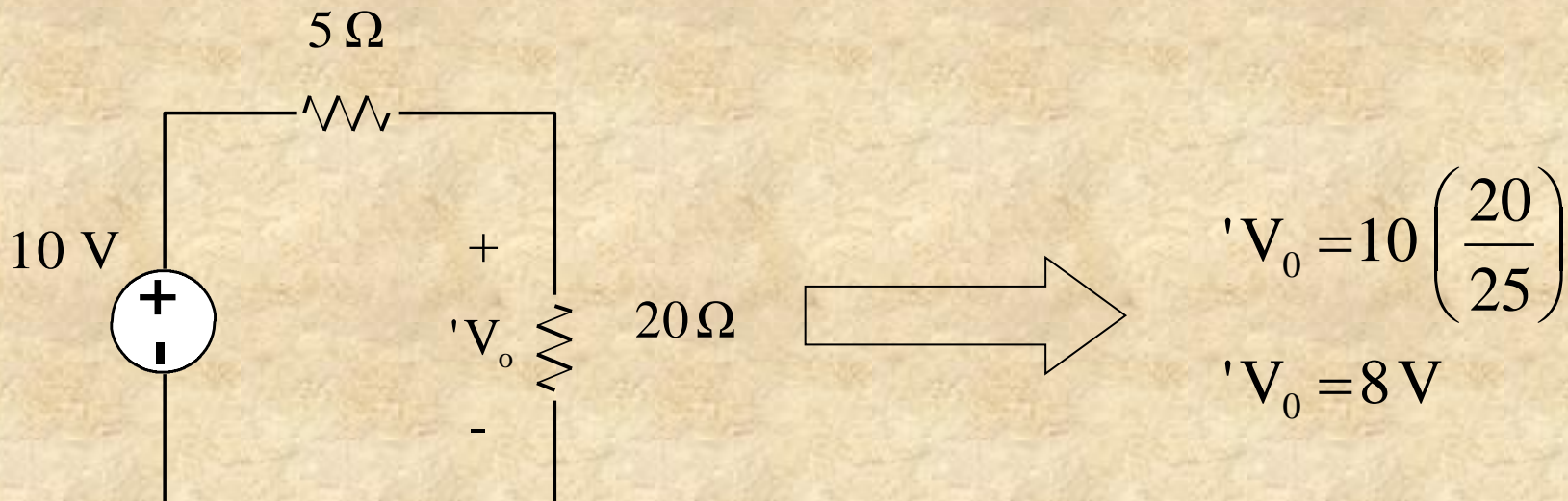


$$V_{\Phi} = 10 (0.4 V_{\Phi}) = 4 V_{\Phi}$$

$$V_{\Phi} = 4 V_{\Phi} \Rightarrow V_{\Phi} = 0$$



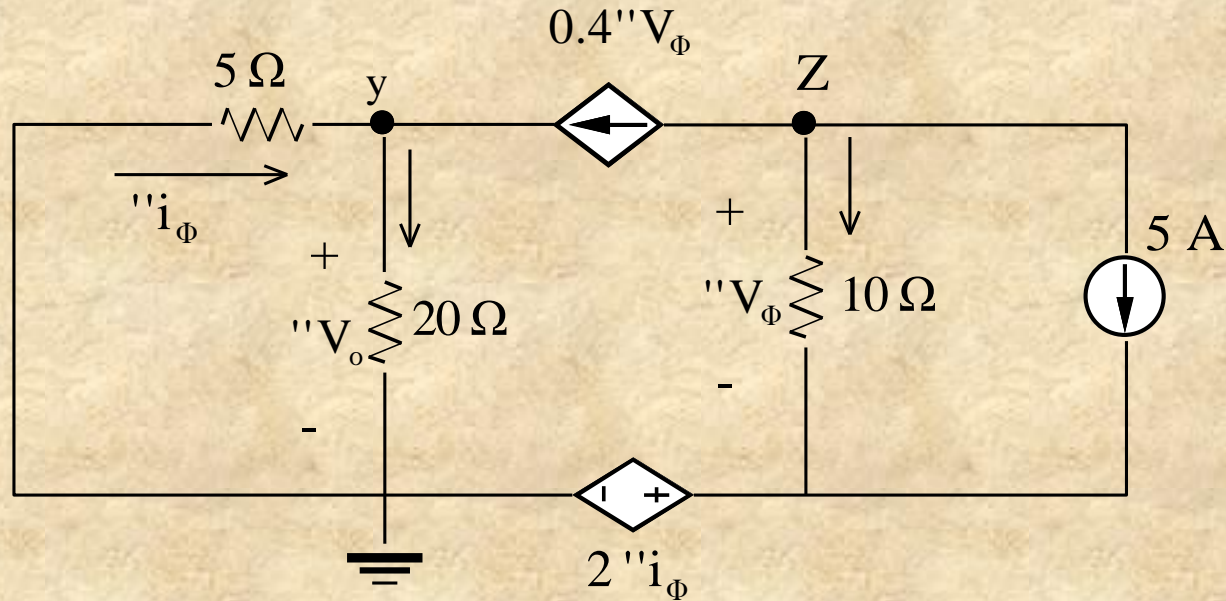
Dependent current source is open



$$V_o = 10 \left( \frac{20}{25} \right)$$

$$V_o = 8 \text{ V}$$

Activate independent current source only:



KCL at node (y):

$$\frac{-V_0}{5} - \frac{V_0}{20} + 0.4 V_\Phi = 0$$

$$-4 V_0 - V_0 + 8 V_\Phi = 0$$

$$-5 V_0 + 8 V_\Phi = 0 \quad \dots\dots(1)$$

$$5 + \frac{V_{\Phi}}{10} + 0.4 V_{\Phi} = 0$$

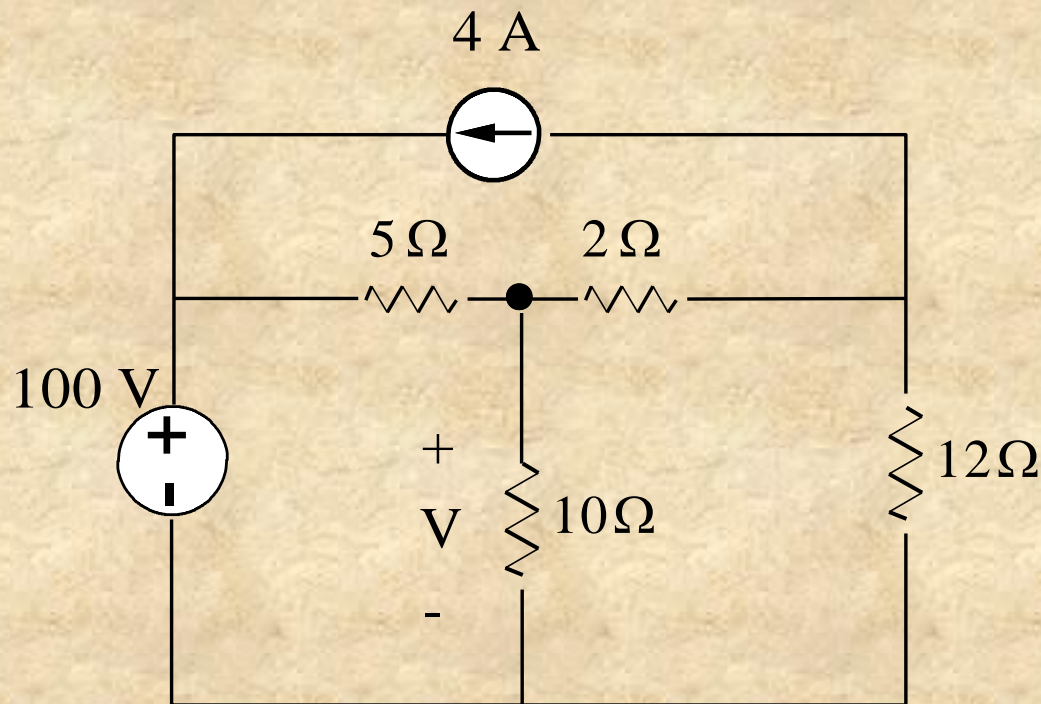
$$0.5 V_{\Phi} = -5 \quad \Rightarrow \quad V_{\Phi} = -10 \text{ V}$$

$$\therefore V_0 = \frac{8}{5} V_{\Phi} = \frac{-80}{5} = -16 \text{ V}$$

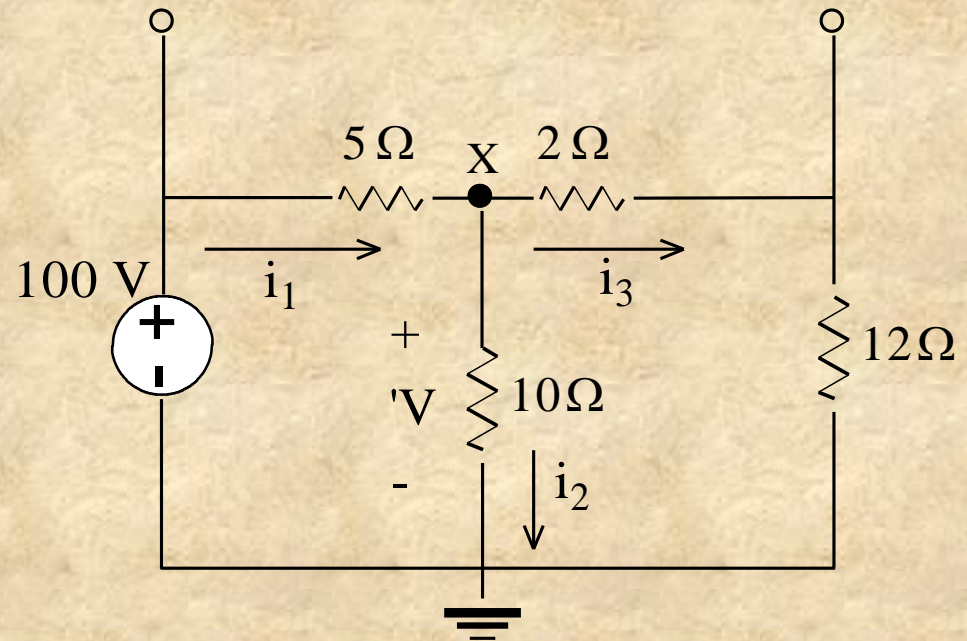
$$V_0 = V_0' + V_0'' = 8 - 16 = -8 \text{ V}$$

## Example:

Use superposition to find  $V$  ?



Consider the independent source only



Apply KCL at node (x) :

$$i_1 - i_2 - i_3 = 0$$

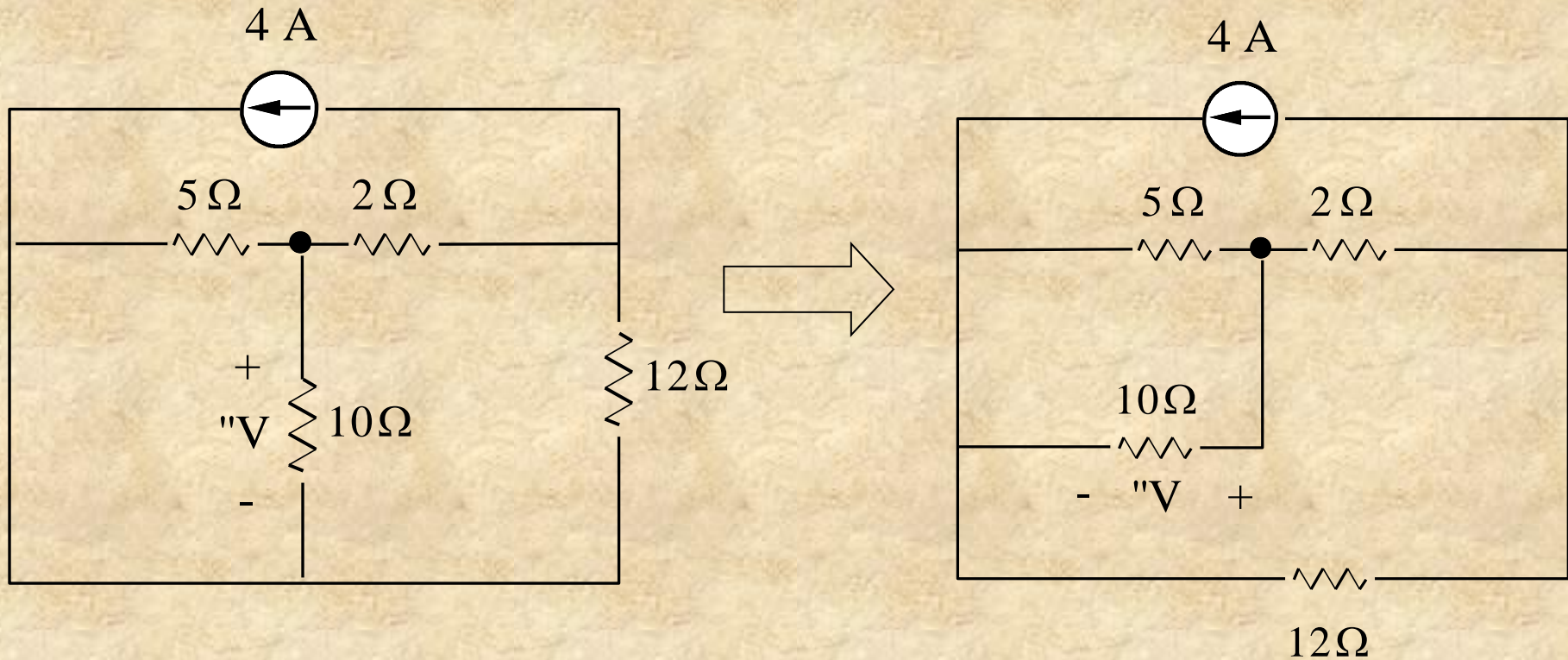
$$\frac{100 - 'V}{5} - \frac{'V}{10} - \frac{'V}{14} = 0$$

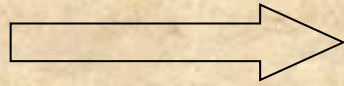
$$22 - \frac{'V}{5} - \frac{'V}{10} - \frac{'V}{14} = 0$$

$$'V \left[ \frac{1}{5} + \frac{1}{10} + \frac{1}{14} \right] = 22 \quad \Rightarrow \quad 'V = 59.23 \text{ V}$$



Consider the independent source only.



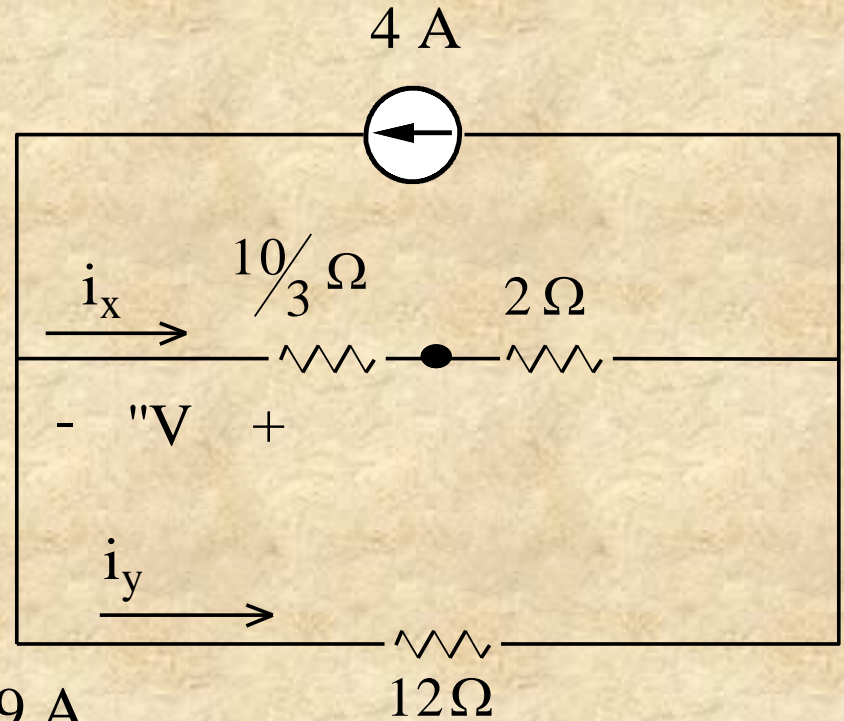


Current divider

$$i_x = 4 \text{ A} \left( \frac{12}{12 + \frac{10}{3} + 2} \right) = 2.769 \text{ A}$$

$$''V = -i_x \left( \frac{10}{3} \right) = -9.23 \text{ V}$$

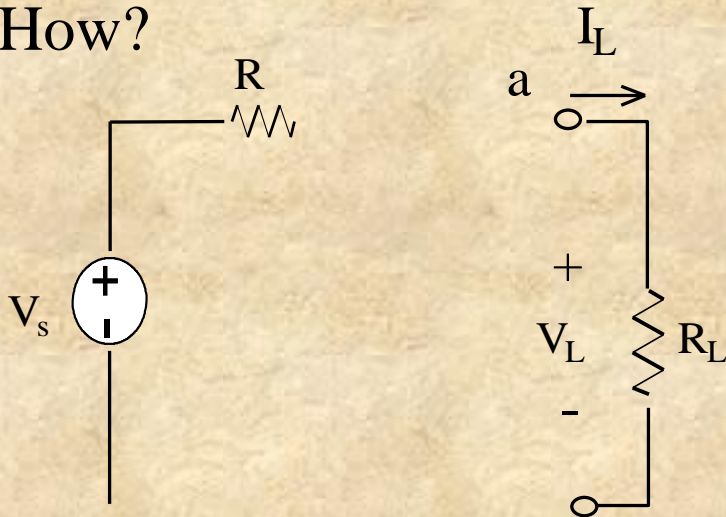
$$V = 'V + ''V = 59.23 - 9.23 = 50 \text{ V}$$



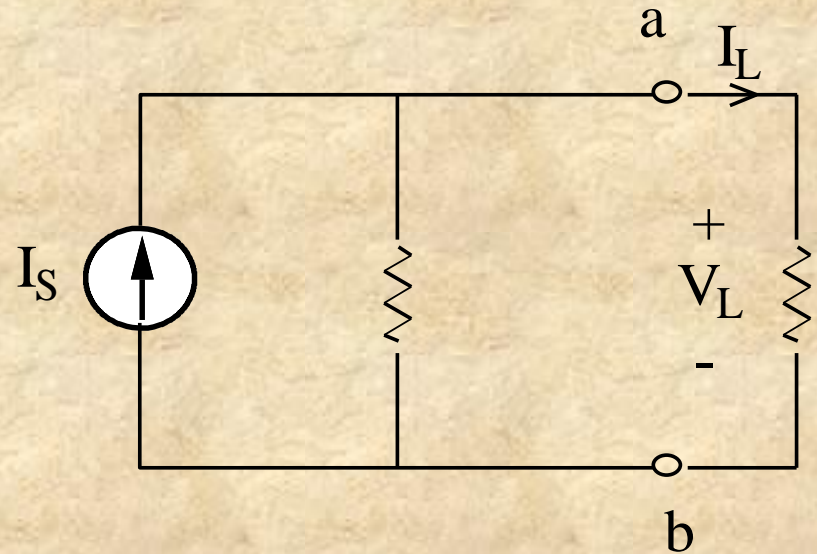
## 2. Source Transformation:

A transformation that allow a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa

How?



KCT 1



KCT 2

We need to find  $I_s$  and  $V_s$  such that  $V_L$  and  $I_L$  is the same in both circuits

In KCT 1 ,

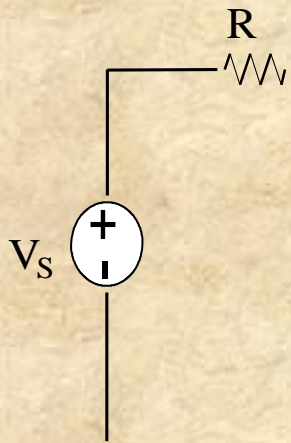
$$I_L = \frac{V_s}{R_L + R}$$

In KCT 2,

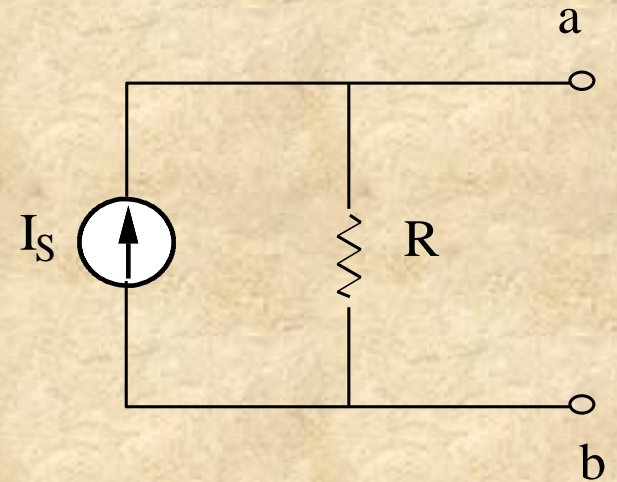
$$I_L = \frac{R}{R_L + R} I_s$$

For  $I_L$  to be the same , we need

$$V_s = R I_s$$



a ○  
b ○



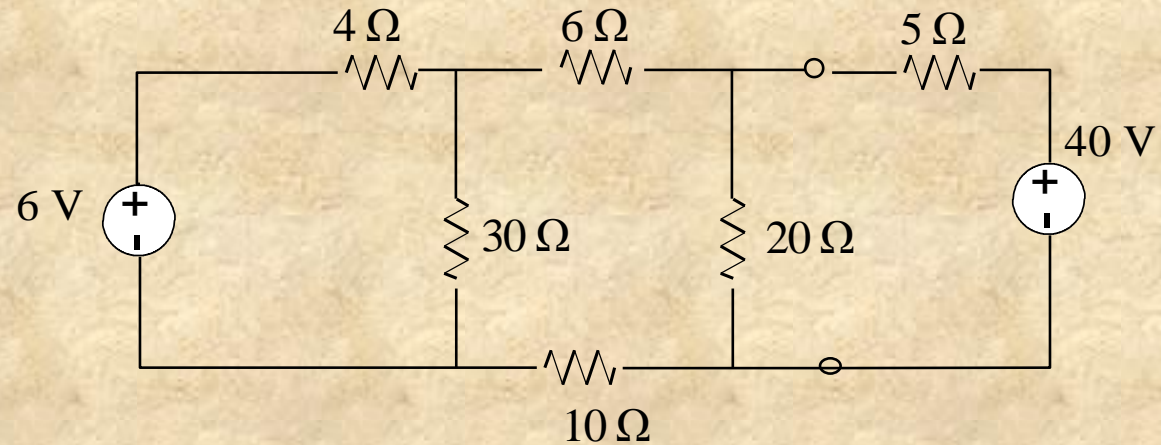
Where

$$V_s = R I_s \quad \text{or} \quad I_s = \frac{V_s}{R}$$

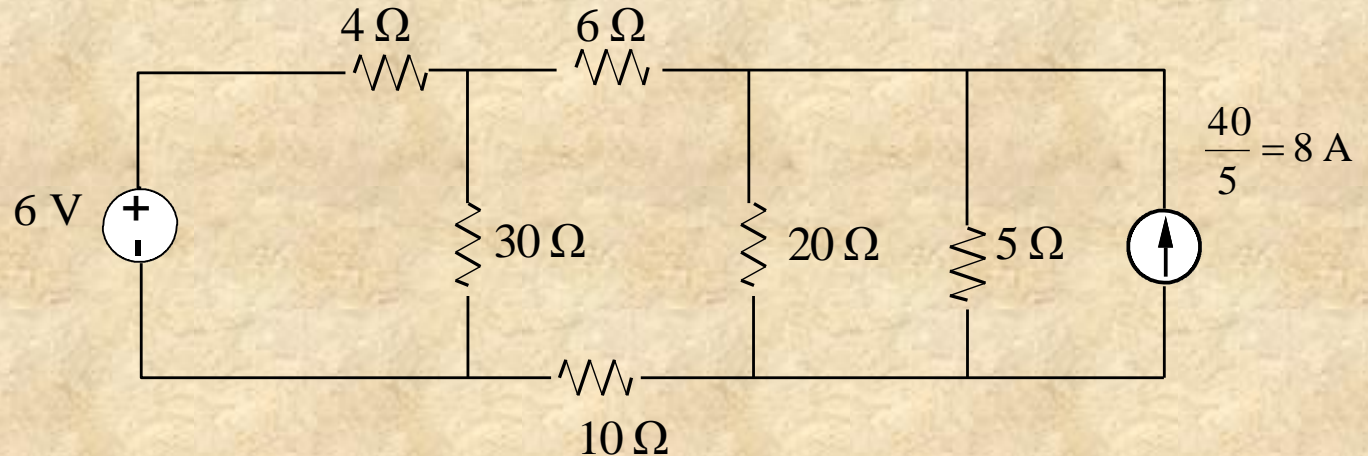


## Example :

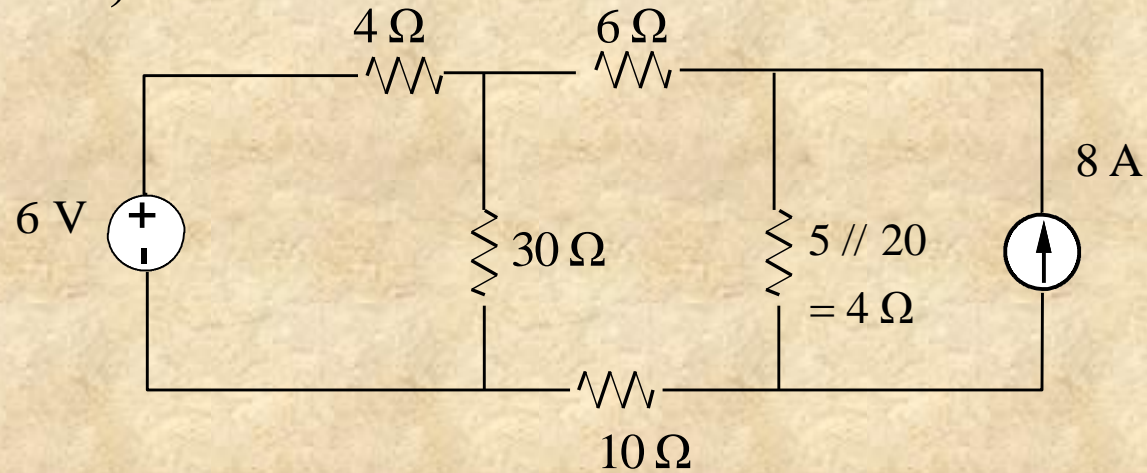
Using source transformation, find the power associated with the 6 V source.



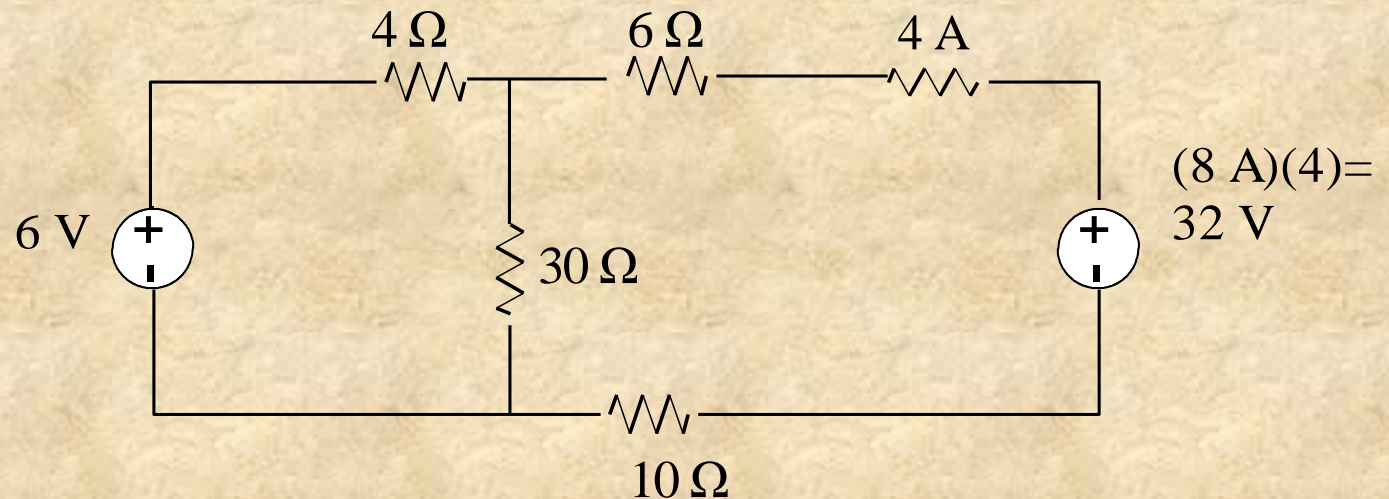
1. Consider the 40 V source in series with (5 Ω)



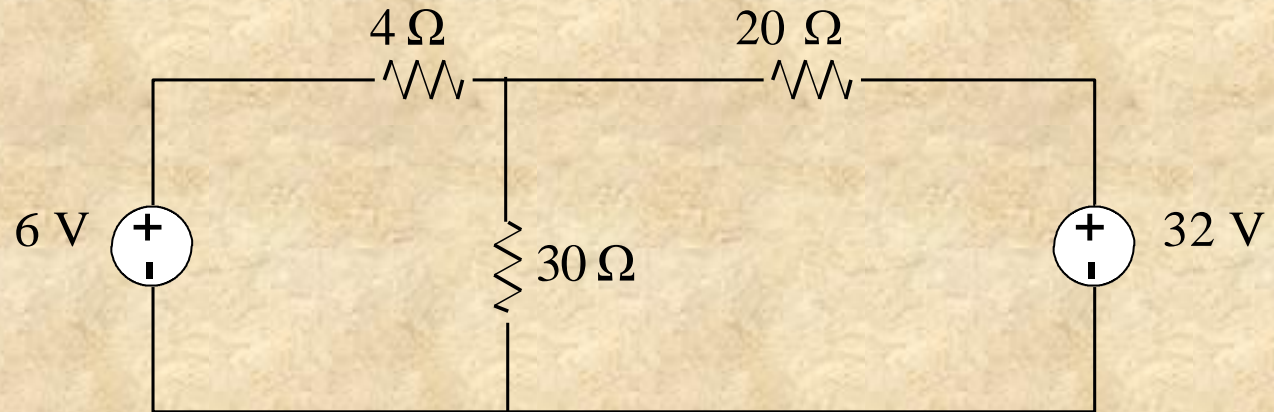
## 2. Take ( $5 // 20 \Omega$ )



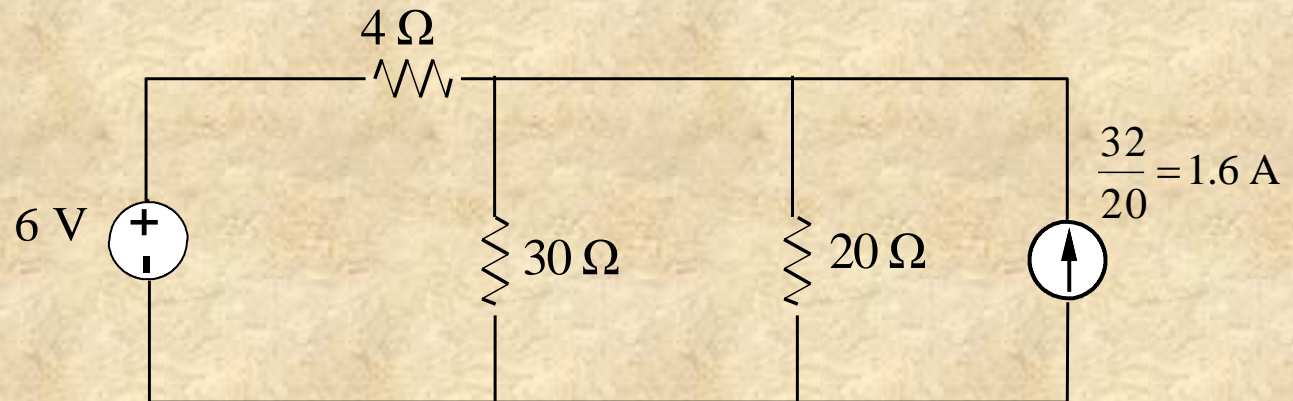
## 3. Consider 8A in parallel with ( $4 \Omega$ )



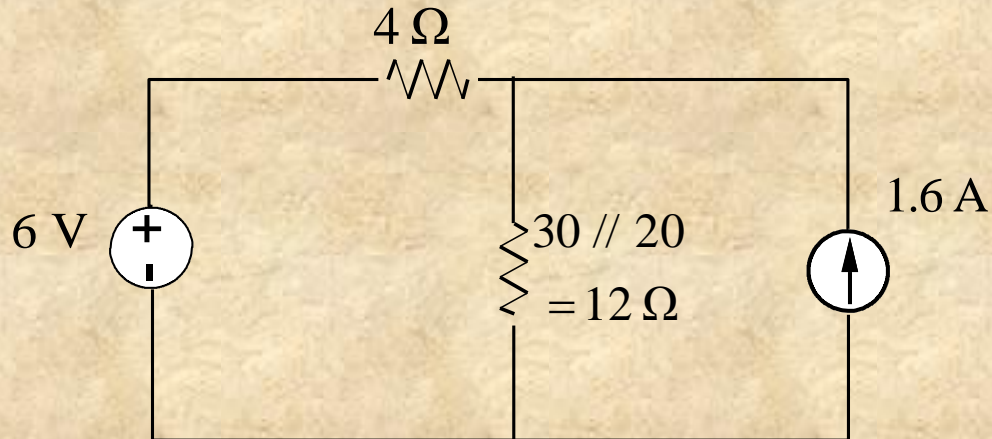
4. Take (4+6+10) in series



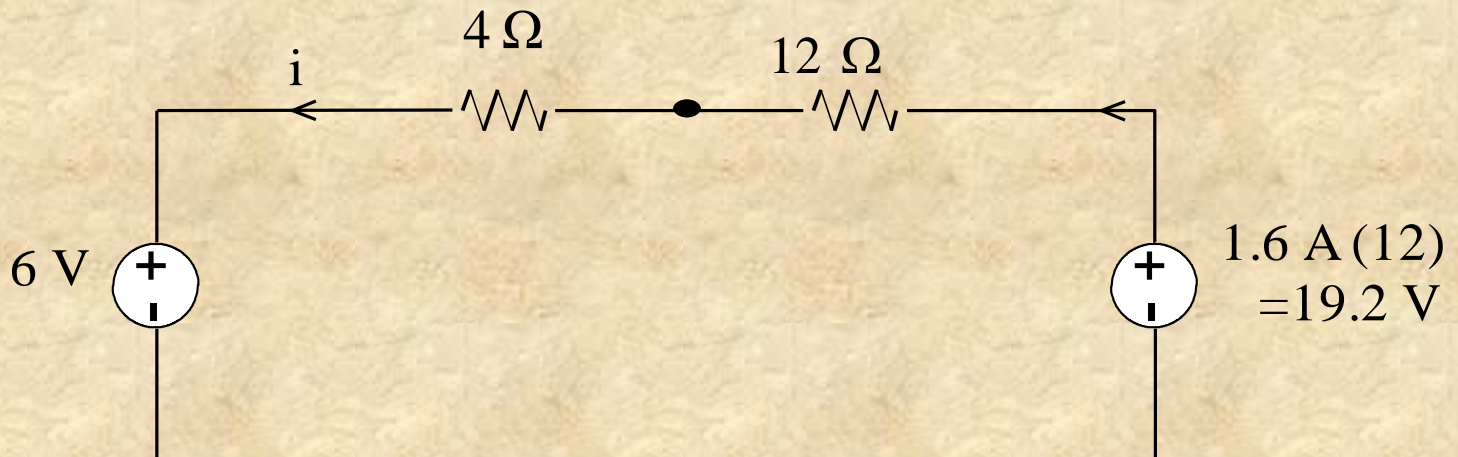
5. Consider 32 V in series with (20Ω)



6. Take ( $30 // 20 \Omega$ )



7. Consider  $1.6 \text{ A}$  in parallel with ( $15 \Omega$ )

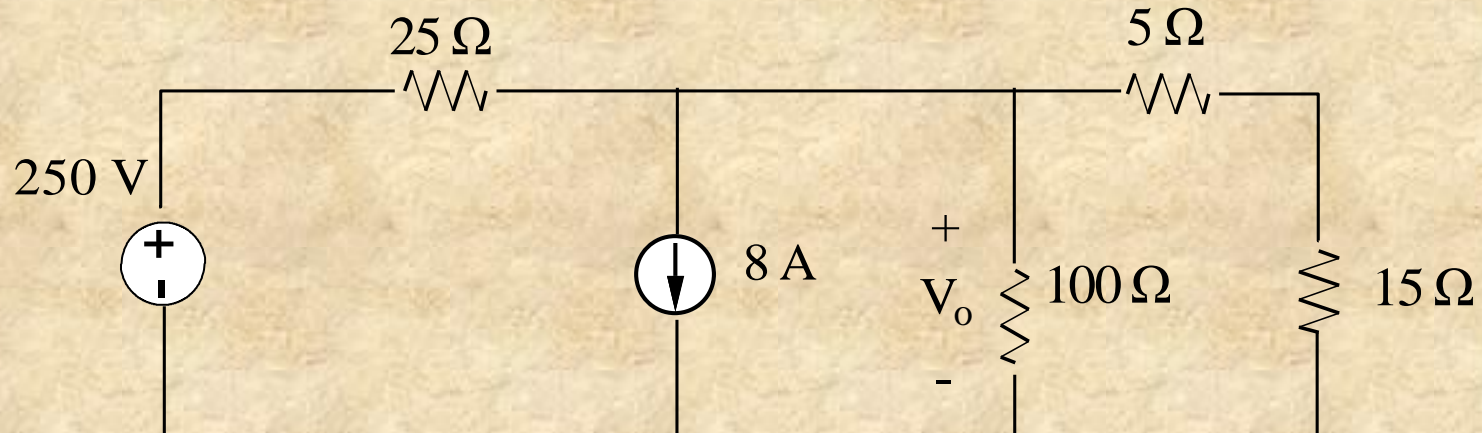


$$i = \frac{19.2 - 6}{4 + 12} = 0.825 \text{ A} \quad \Rightarrow \quad P_{6V} = v i = 6 (0.825)$$

$$P_{6V} = 4.95 \text{ W (absorbing)}$$

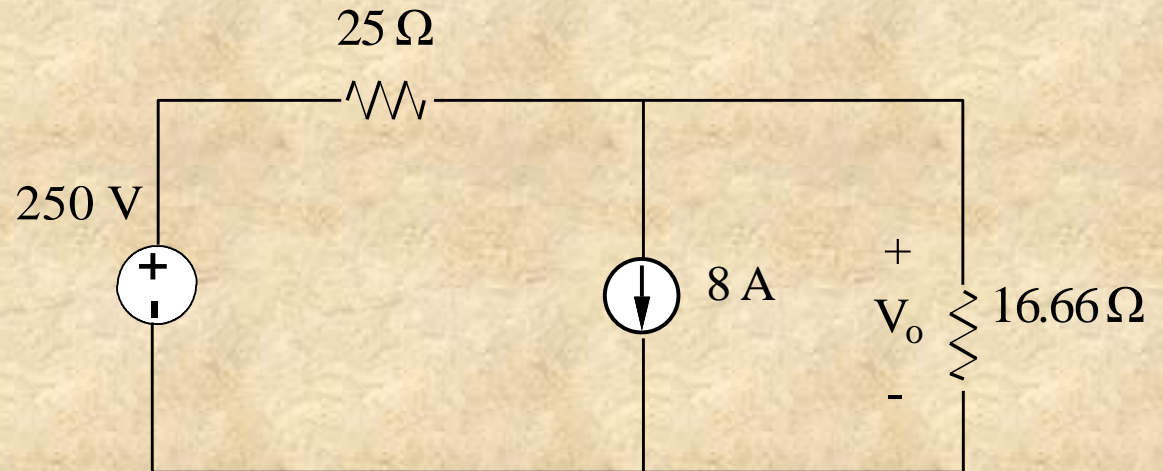
### Example :

Use source transformation to find  $V_0$

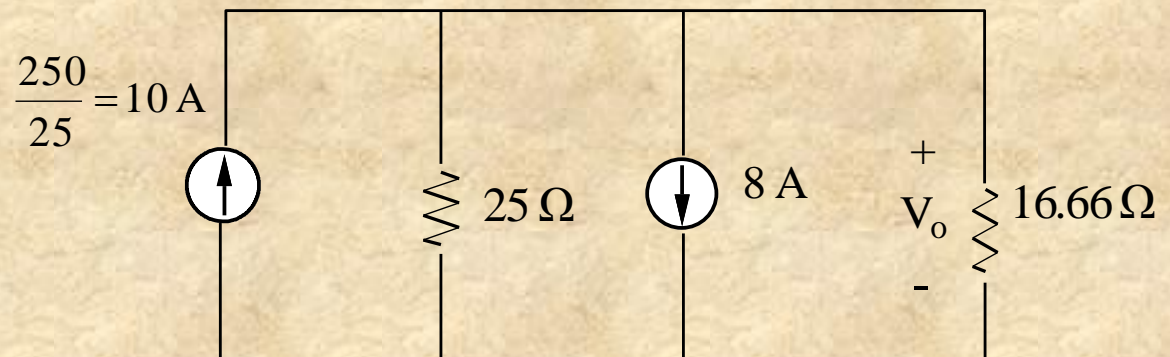




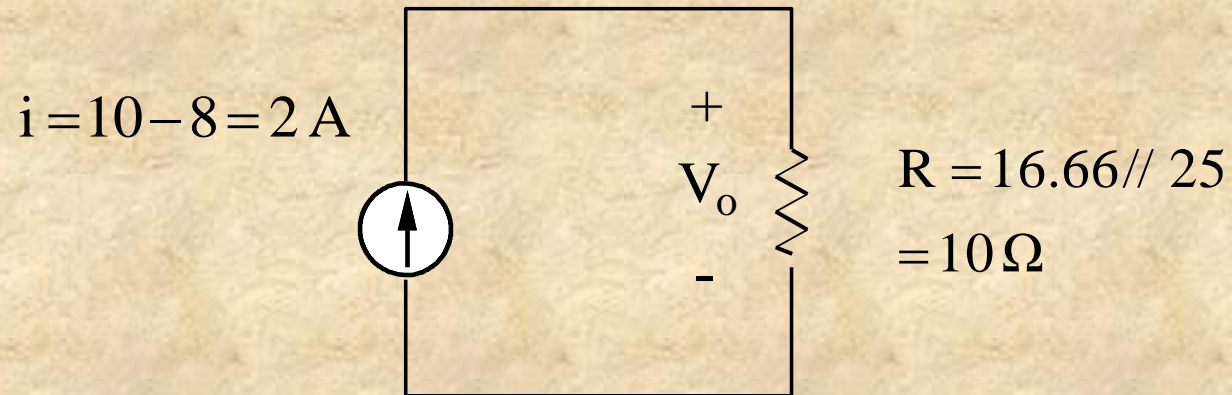
1. Take  $(5//15)//100 = 6.66 \Omega$



2. Consider (250 V) in series with (25 Ω)



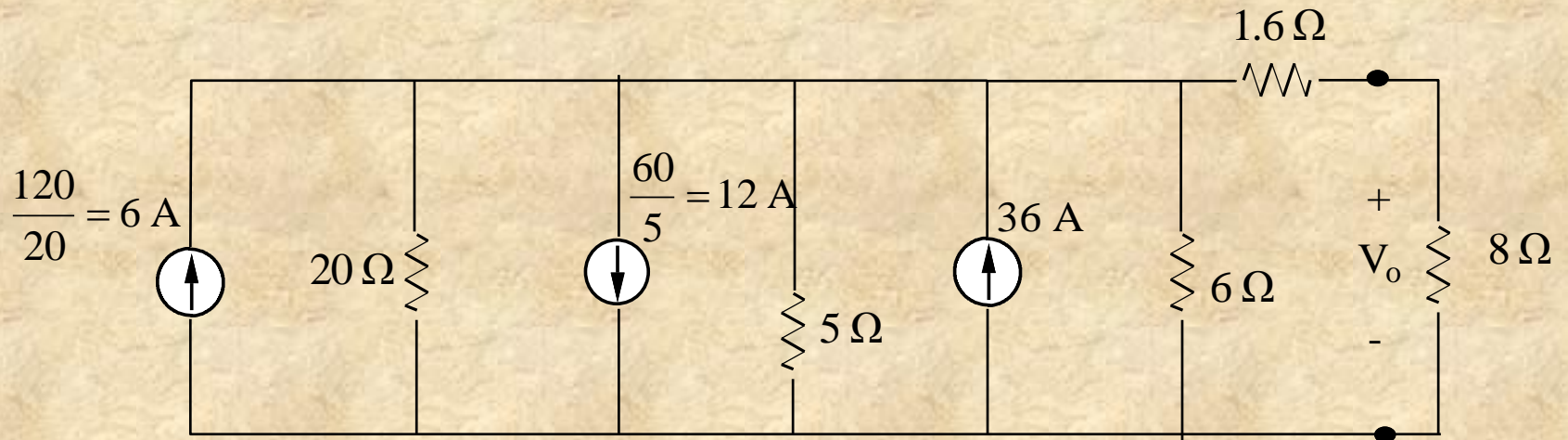
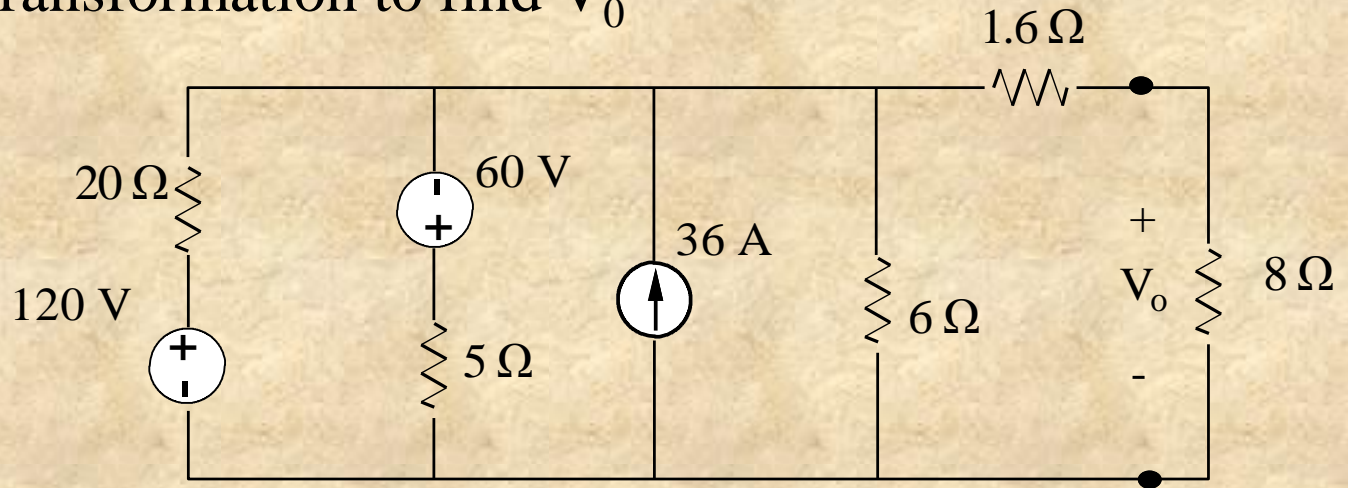
### 3. Find equivalent

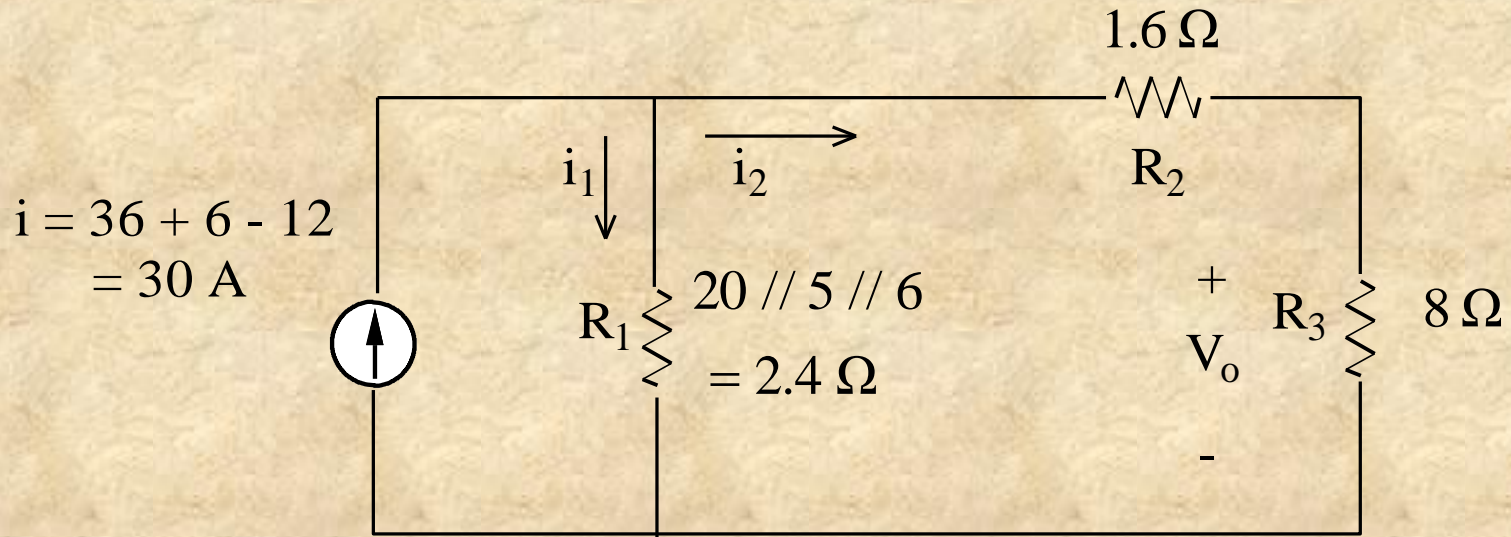


$$V_o = i R = (2 \text{ A}) (10 \Omega) = 20 \text{ V}$$

## Example:

Use source transformation to find  $V_0$

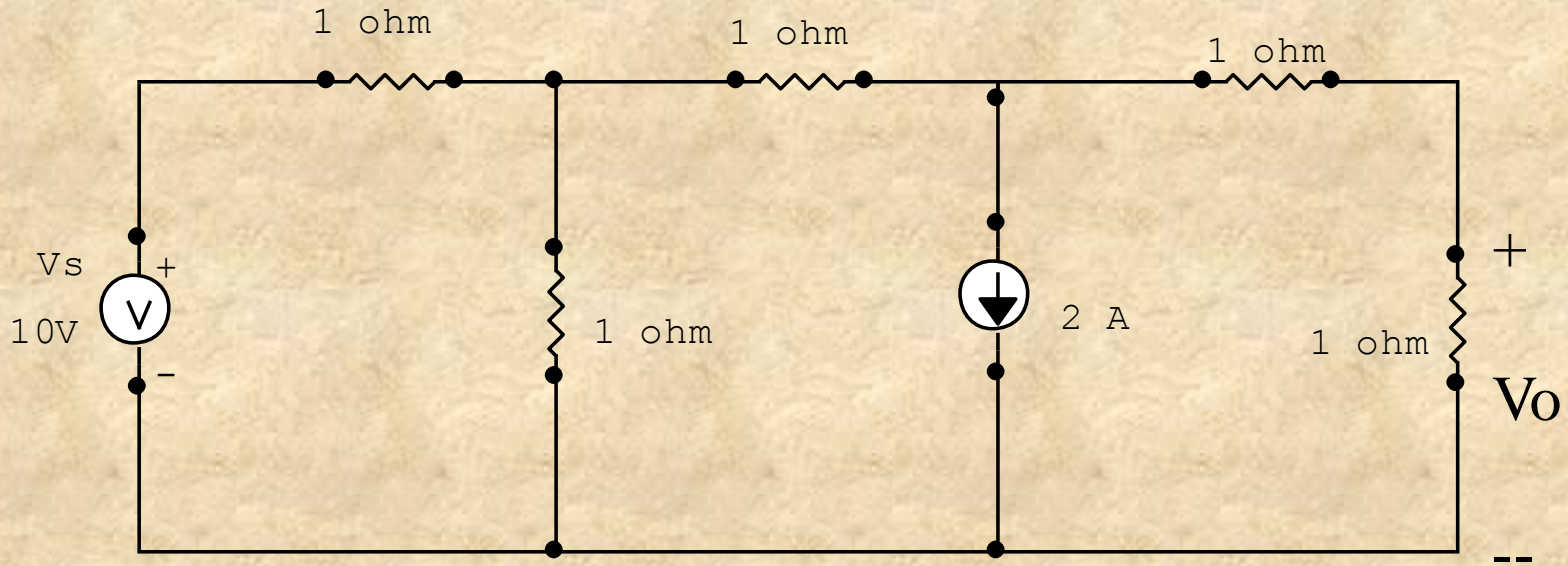




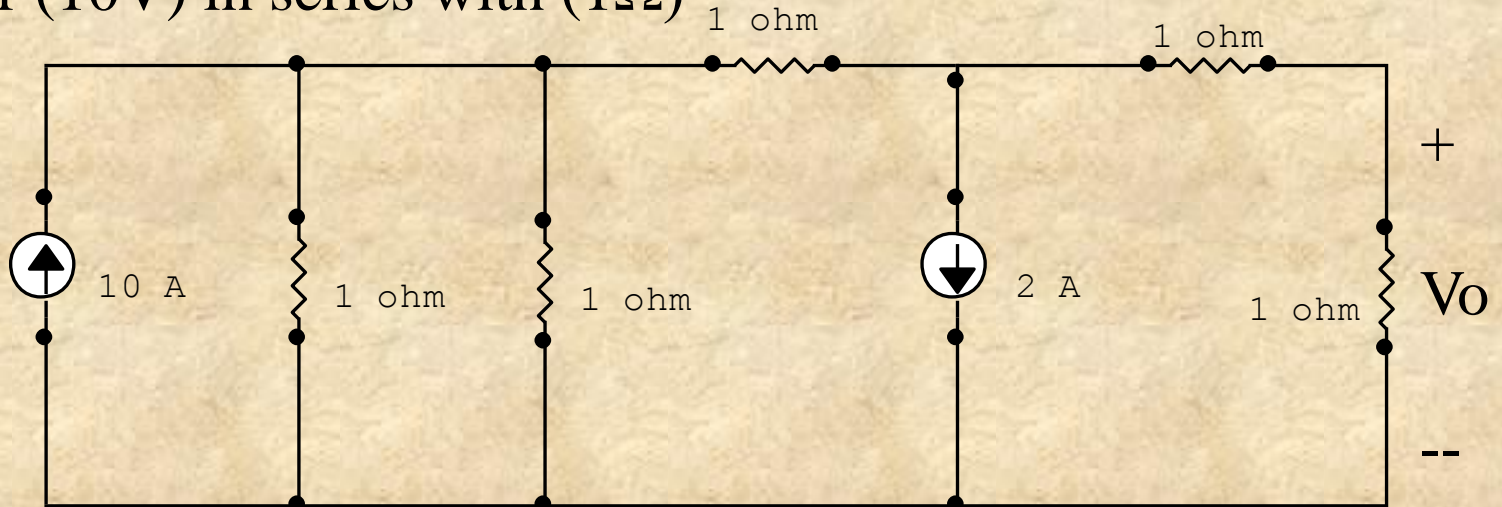
$$i_2 = \frac{R_1}{R_1 + R_2 + R_3} i = \frac{2.4}{2.4 + 1.6 + 8} (30) = 6 \text{ A}$$

$$V_0 = i_2 R_3 = (6 \text{ A}) (8 \Omega) = 48 \text{ V}$$

**Example :** Use source transformation to find  $V_0$ ?

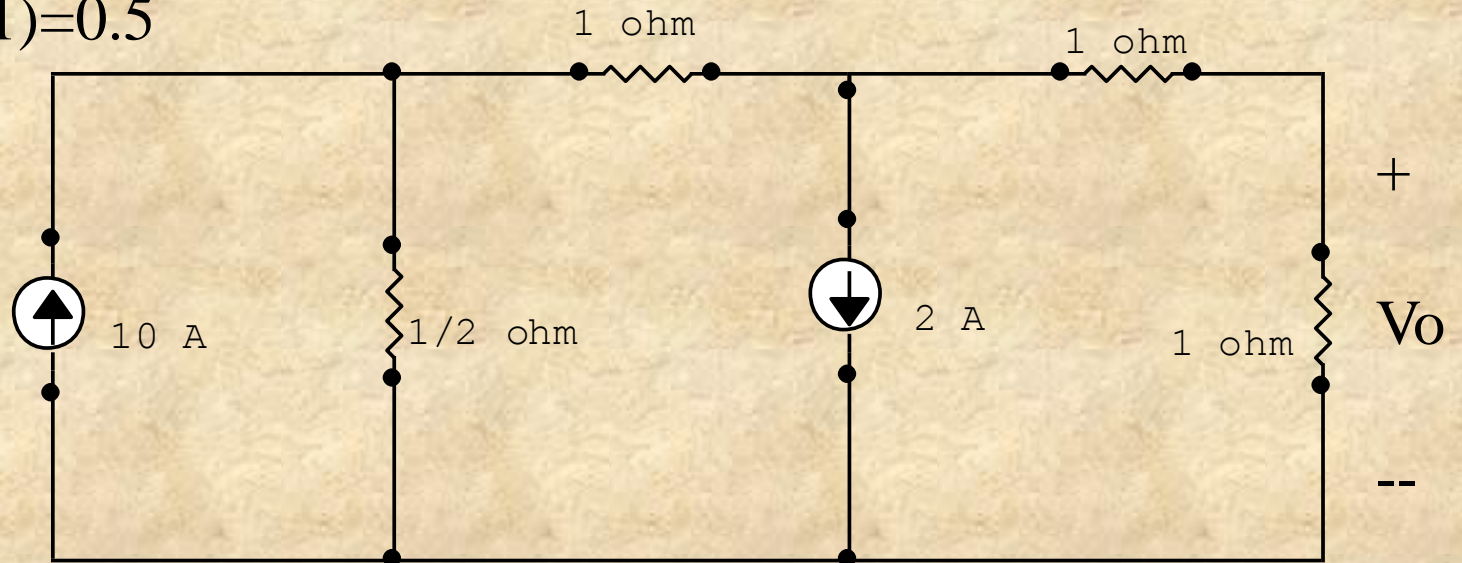


Consider (10V) in series with ( $1\Omega$ )

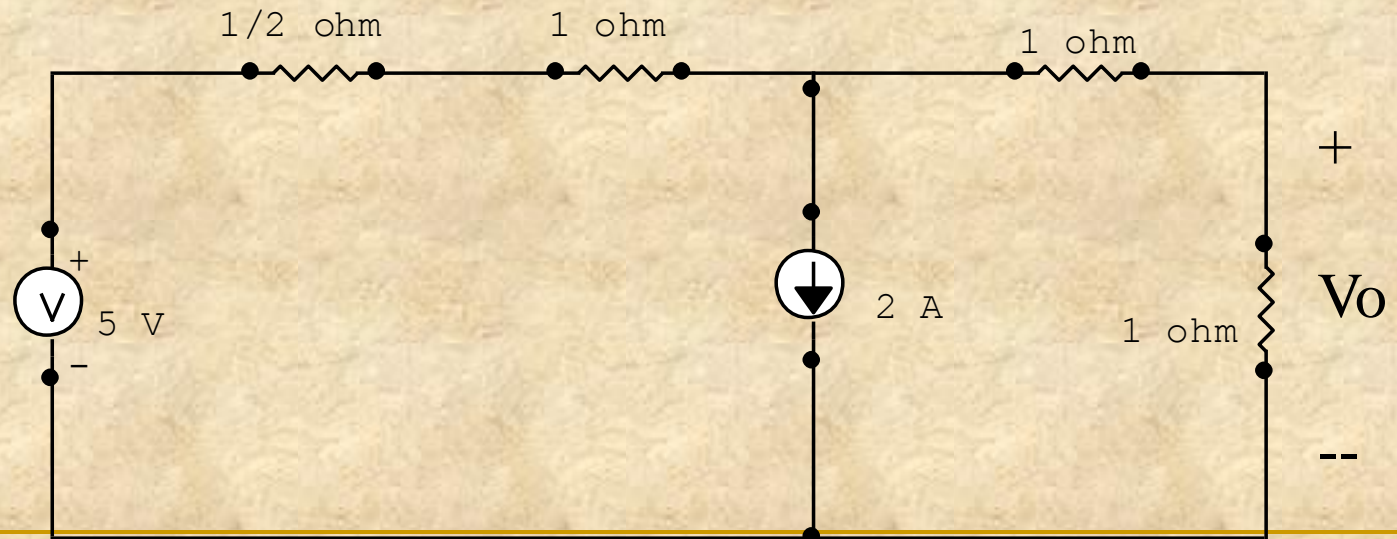




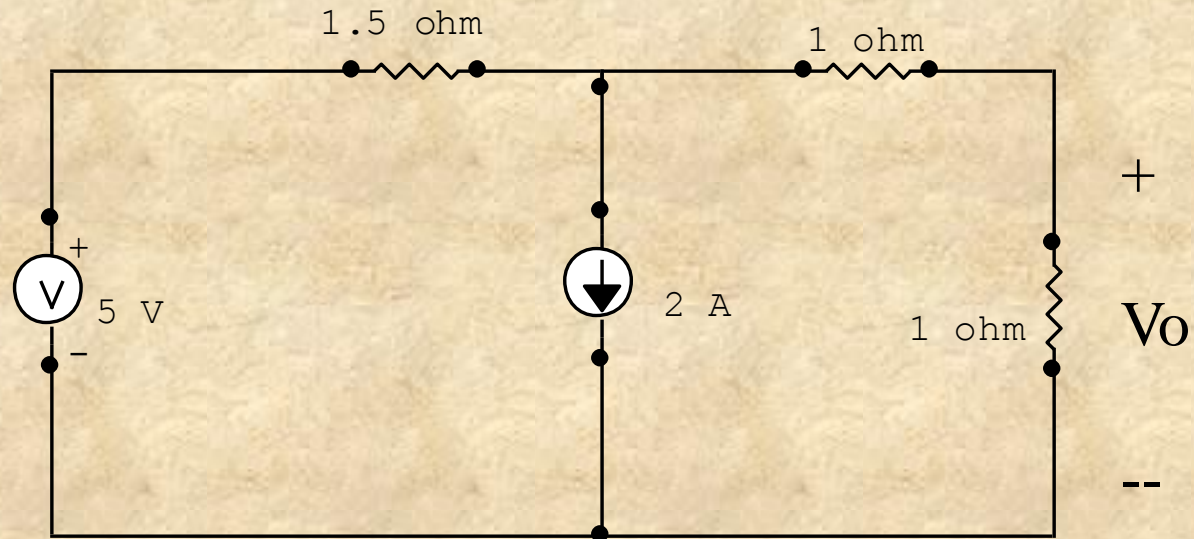
Take  $(1//1)=0.5$



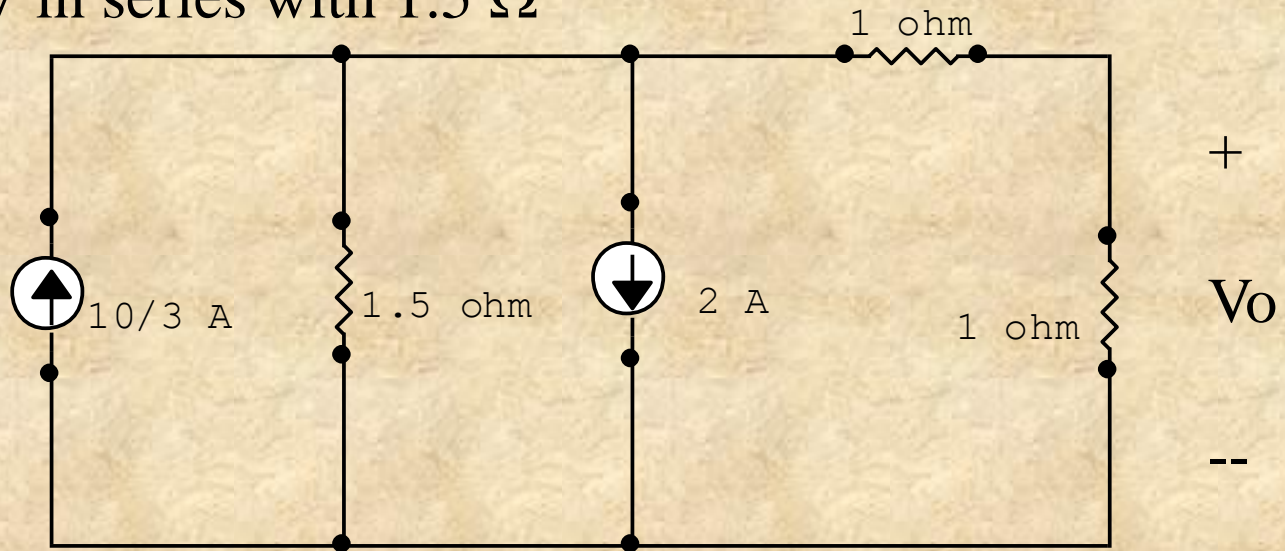
Consider  $(10A)$  in parallel with  $(0.5 \Omega)$



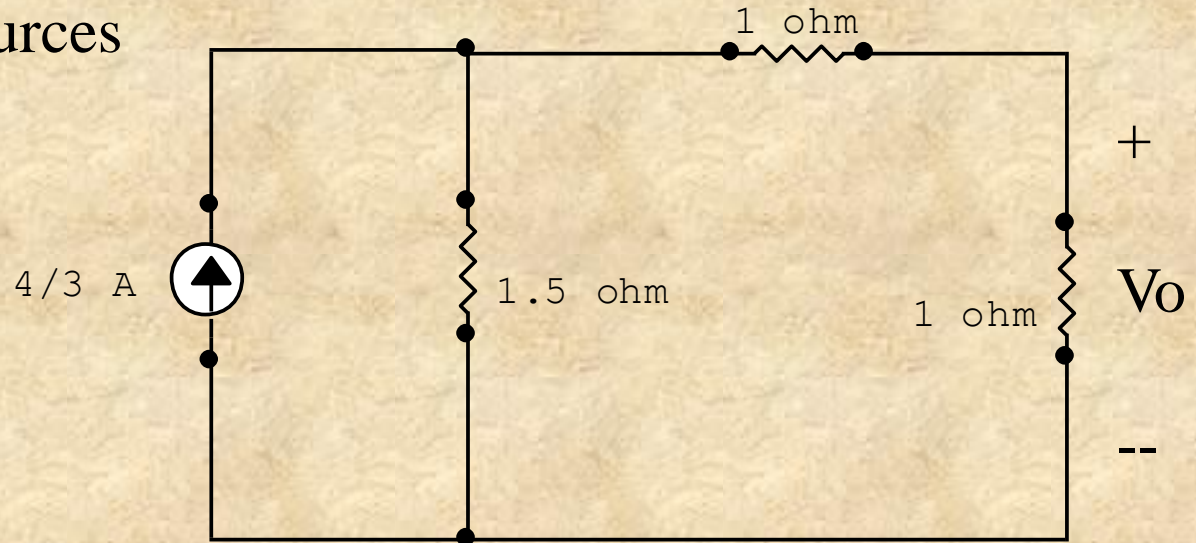
Take  $0.5\Omega$  in series with  $1\Omega$



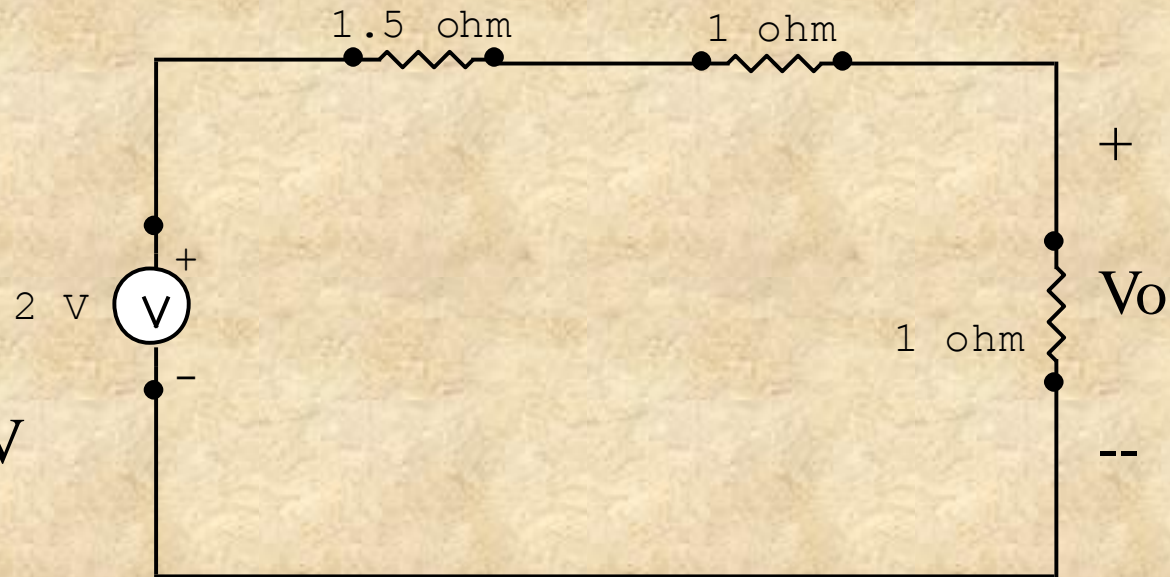
Consider  $5\text{ V}$  in series with  $1.5\ \Omega$



Add the current sources



7. Take  $(4/3A)$  in parallel with  $(3/2 \Omega)$

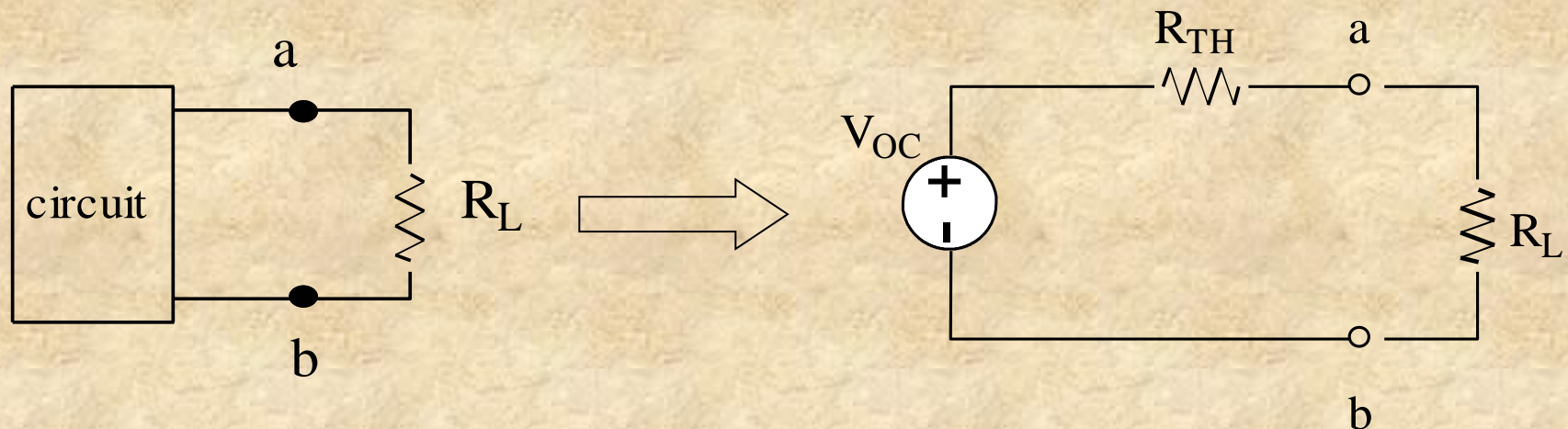


$$V_0 = \frac{1}{1+1+1.5} (2) = 4/7 \text{ V}$$

# Thevenin and Norton Theorems

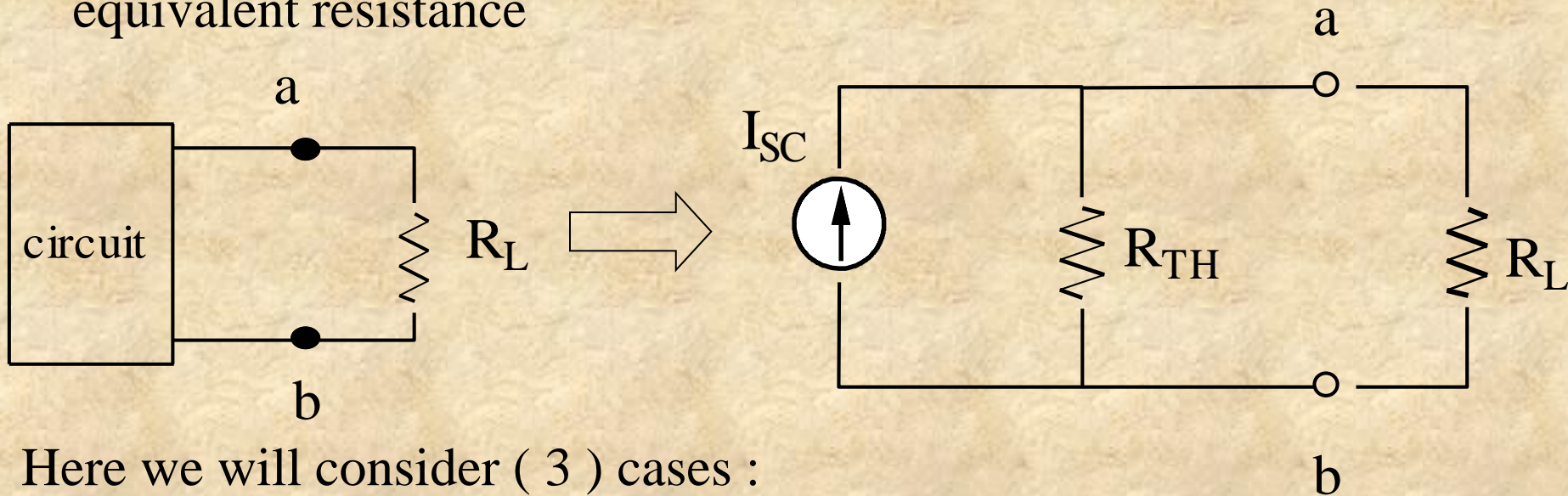
## Thevenin Theorem:

A portion of the circuit at a pair of nodes can be replaced by a voltage source  $V_{oc}$  in series with a resistor  $R_{TH}$ , where  $V_{oc}$  is the open circuit voltage and  $R_{TH}$  is the Thevenin's equivalent resistance obtained by considering the open circuit with all independent sources made zero



## Norton Theorem :

A portion of the circuit at pair of nodes can be replaced by a current source  $I_{sc}$  in a parallel with a resistor  $R_{TH}$ .  $I_{sc}$  is the short circuit current at the terminals, and  $R_{TH}$  is the Thevenin's equivalent resistance



Here we will consider ( 3 ) cases :

1. Circuit containing only independent sources.
2. Circuit containing only dependent sources.
3. Circuit containing both independent and dependent sources.



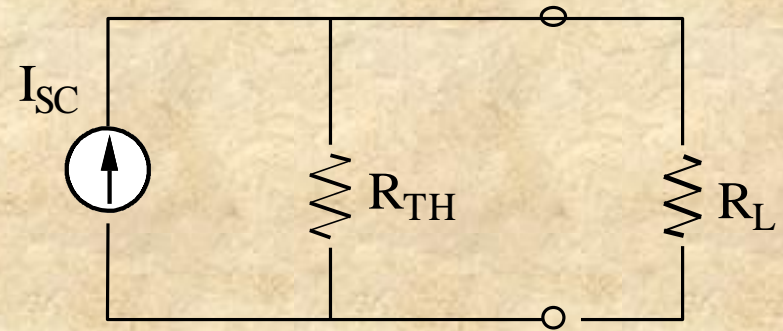
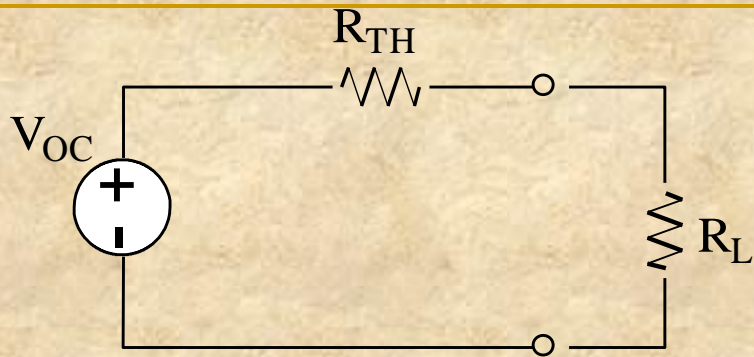
## Case (1): Circuit containing only independent sources:

- **Procedure of Thevenin's Theorem:**

- a. Find the open circuit voltage at the terminals ,  $V_{oc}$ .
- b. Find the Thevenin's equivalent resistance,  $R_{TH}$  at the terminals when all independent sources are zero:
  - Replacing independent voltage sources by short circuit
  - Replacing independent current sources by open circuit
- c. Reconnect the load to the Thevenin equivalent circuit

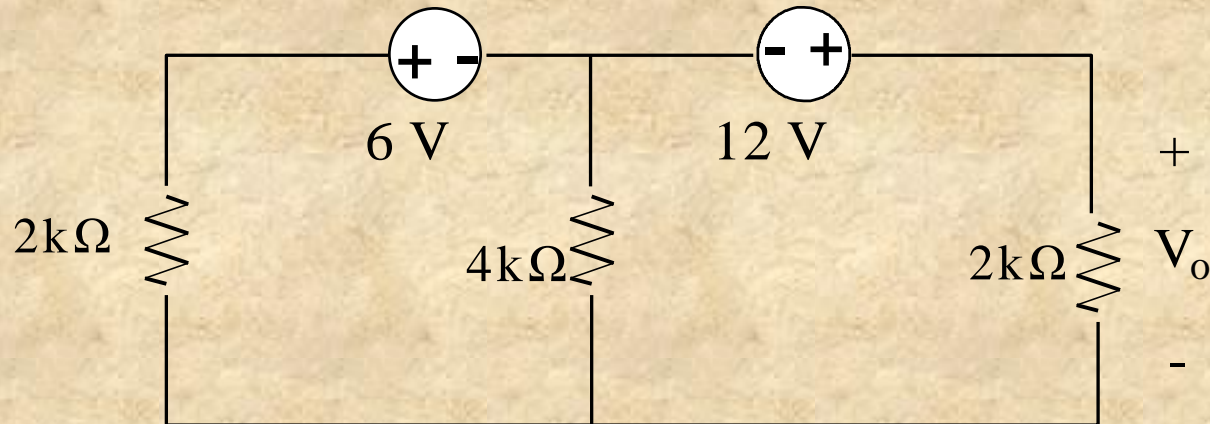
- **Procedure of Norton's Theorem:**

- a. Find the short circuit current at the terminals,  $I_{sc}$ .
- b. Find Thevenin's equivalent resistance,  $R_{TH}$  (as before).
- c. Reconnect the load to Norton's equivalent circuit.



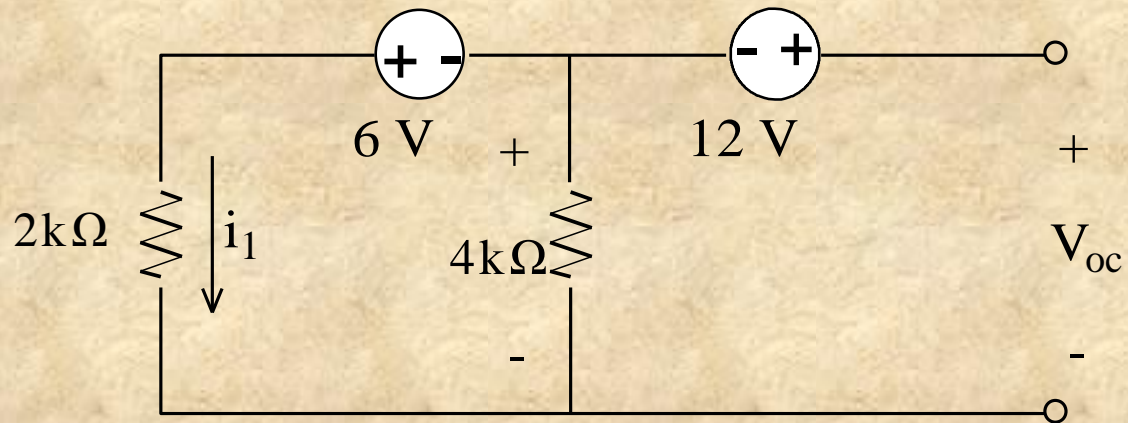
**Example :**

Use Thevenin's and Norton Theorms to find  $V_0$



**Using Thevenin Theorm:**

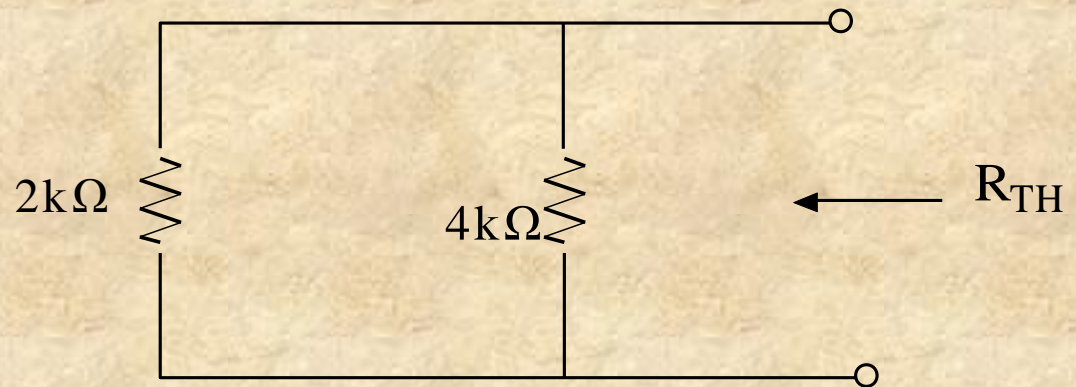
First find  $V_{OC}$ :



$$i_1 = \frac{6 \text{ V}}{2 \text{ k} + 4 \text{ k}} = 1 \text{ mA} \quad \Rightarrow \quad V_{4\text{k}\Omega} = i_1 (4\text{k}) = -4 \text{ V}$$

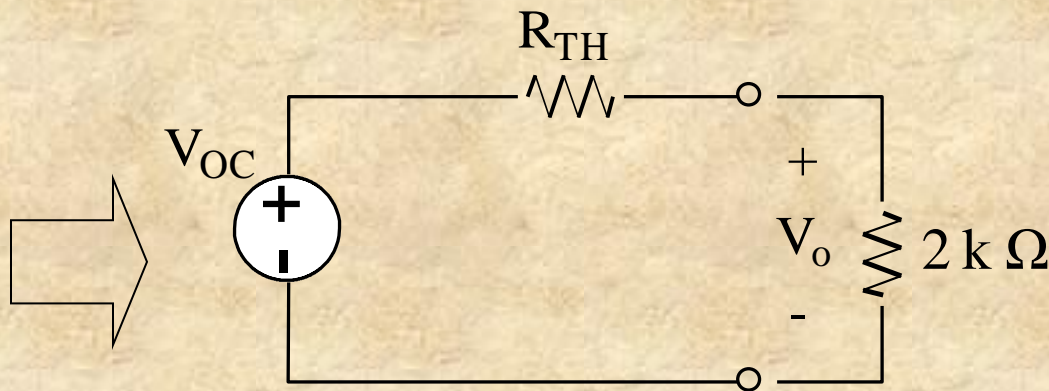
$$V_{OC} = 12 \text{ V} - 4 \text{ V} = 8 \text{ V}$$

Second, find  $R_{TH}$



$$R_{TH} = 2\text{k} // 4\text{k} = \frac{4}{3} \text{ k}\Omega$$

Thevenin equivalent circuit is

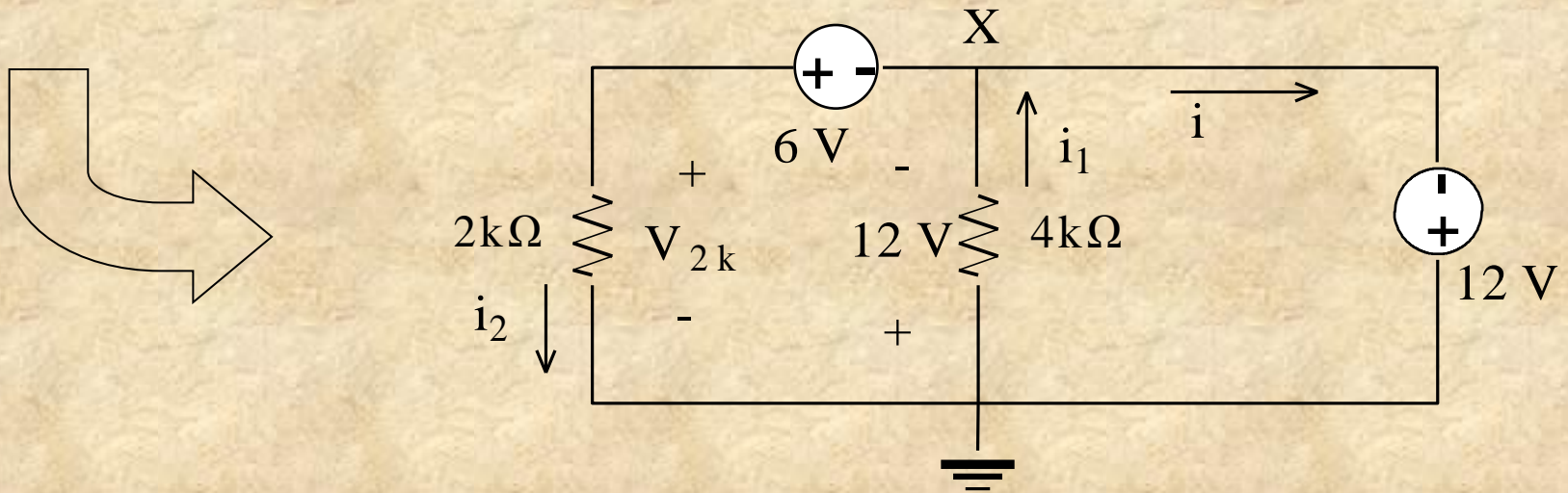
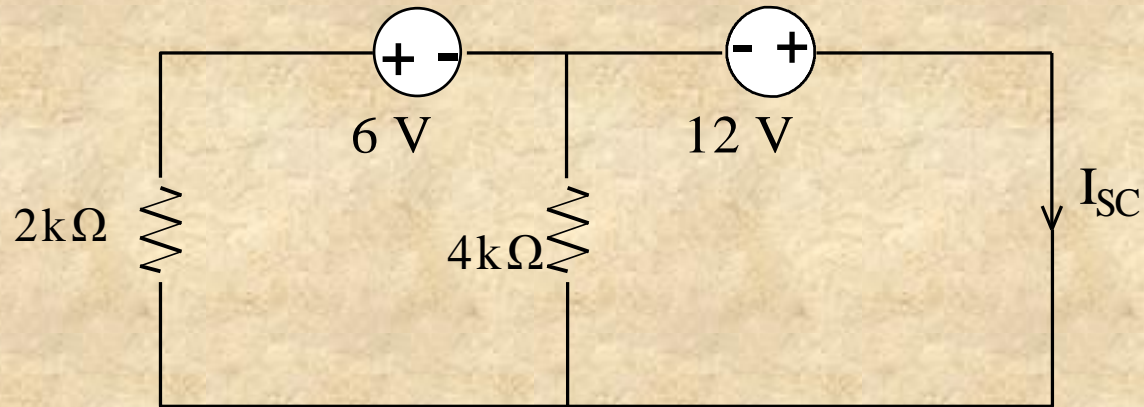


$$V_0 = \frac{2\text{ k}\Omega}{2\text{ k} + R_{TH}} V_{oc}$$
$$= \frac{2\text{ k}}{\frac{10}{3}\text{ k}} (8\text{ V})$$

$$V_0 = 4.8\text{ V}$$

# Using Norton Theorem

First find  $I_{sc}$





$$i_1 = \frac{12 \text{ V}}{4 \text{ k}} = 3 \text{ m A}$$

KVL around outer loop:

$$12 - 6 + V_{2k} = 0 \quad \Rightarrow \quad V_{2k} = -6 \text{ V}$$

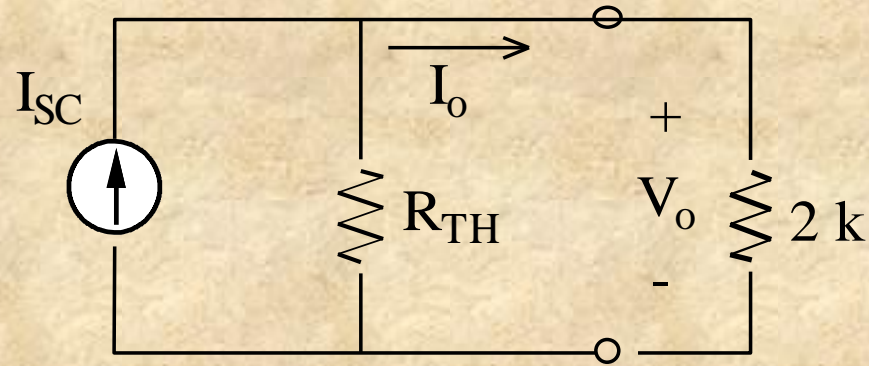
$$i_2 = \frac{V_{2k}}{2 \text{ k}} = \frac{-6}{2 \text{ k}} = -3 \text{ m A}$$

KCL at x :

$$i_1 - i_2 - i = 0$$

$$3 \text{ m} + 3 \text{ m} - i = 0 \quad \Rightarrow \quad i = 6 \text{ m A} \quad \Rightarrow \quad I_{sc} = 6 \text{ m A}$$

$R_{TH}$  is the same as before:

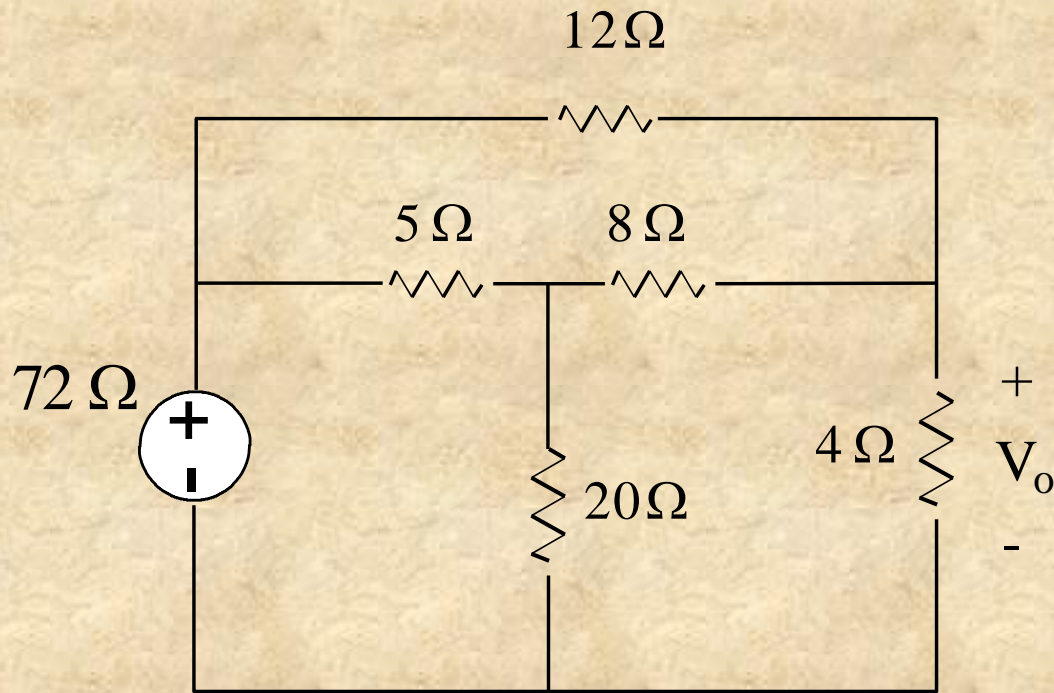


$$I_o = \frac{R_{TH}}{R_{TH} + 2\text{ k}} (I_{sc}) = \frac{\frac{4}{3}\text{ k}}{\frac{4}{3}\text{ k} + 2\text{ k}} (6\text{ m}) = 2.4\text{ m A}$$

$$V_o = I_o (2\text{ k}) = (2.4\text{ m}) (2\text{ k}) = 4.8\text{ V}$$

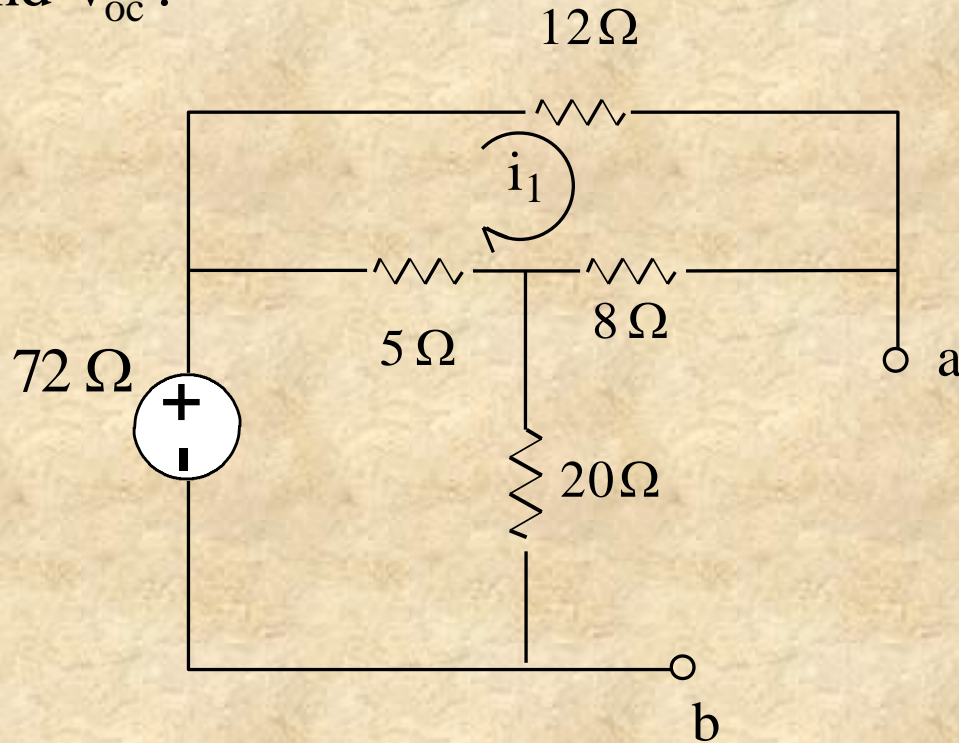
**Example :**

Use Thevenin and Norton to find  $V_0$



Using Thevenin Theorem:

1. Find  $V_{oc}$  :



KVL around the upper loop :

$$12 i_1 + 8 i_1 + 5 (i_1 - i_2) = 0$$

$$25 i_1 - 5 i_2 = 0 \quad \dots\dots(1)$$

KCL around lower loop :

$$5 (i_2 - i_1) + 20 i_2 = 72$$

$$-5i_1 + 25 i_2 = 72 \quad \dots\dots(2)$$

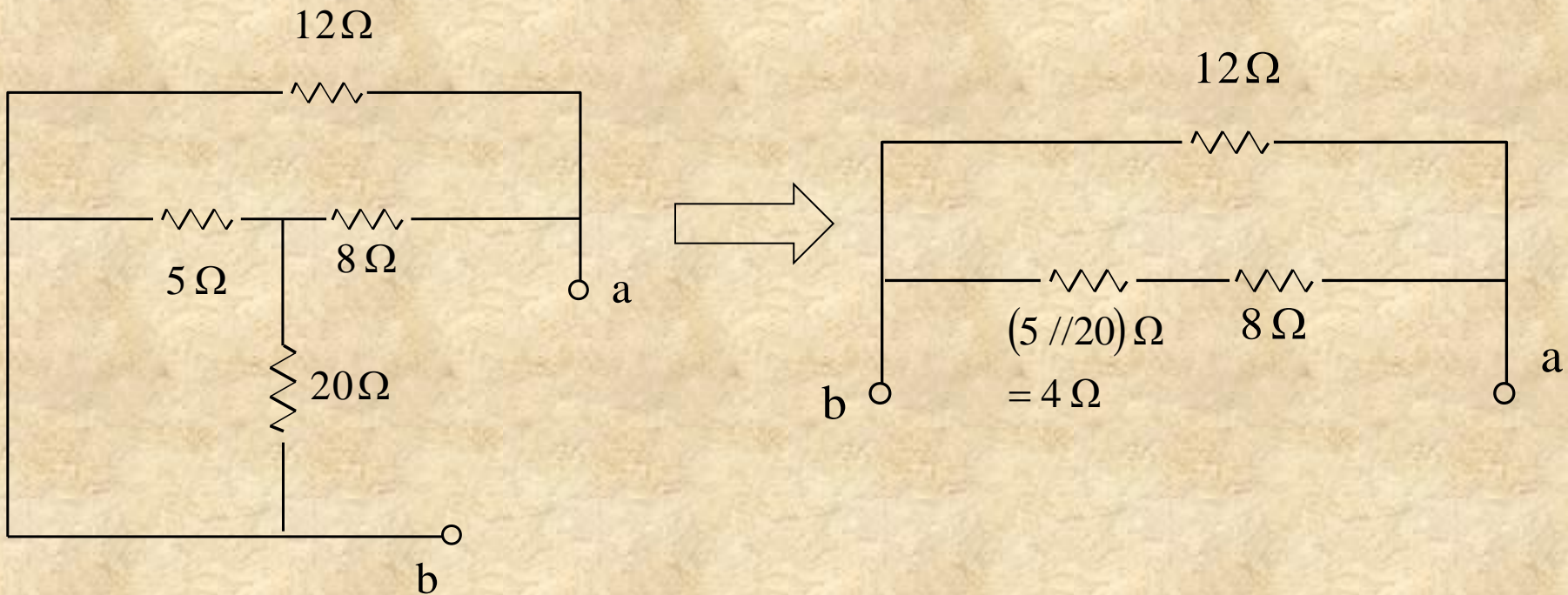
$$i_1 = 0.6 \text{ A}, \quad i_2 = 3 \text{ A}$$

$$\begin{aligned} V_{oc} &= 8 i_1 + 20 i_2 \\ &= 8 (0.6) + 20 (3) \end{aligned}$$

$$V_{oc} = 64.8 \text{ V}$$

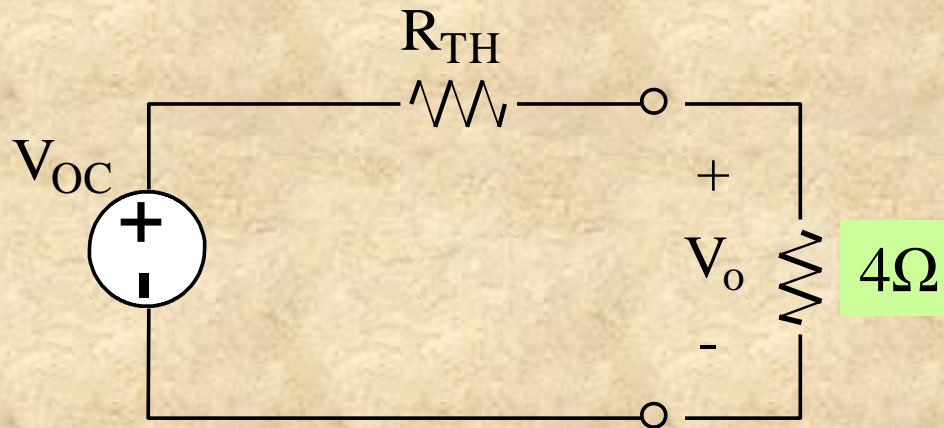


## 2. Find $R_{TH}$



$$R_{TH} = (8+4) // 12 = 12 // 12 = 6\Omega$$

### 3. Reconnect the load :



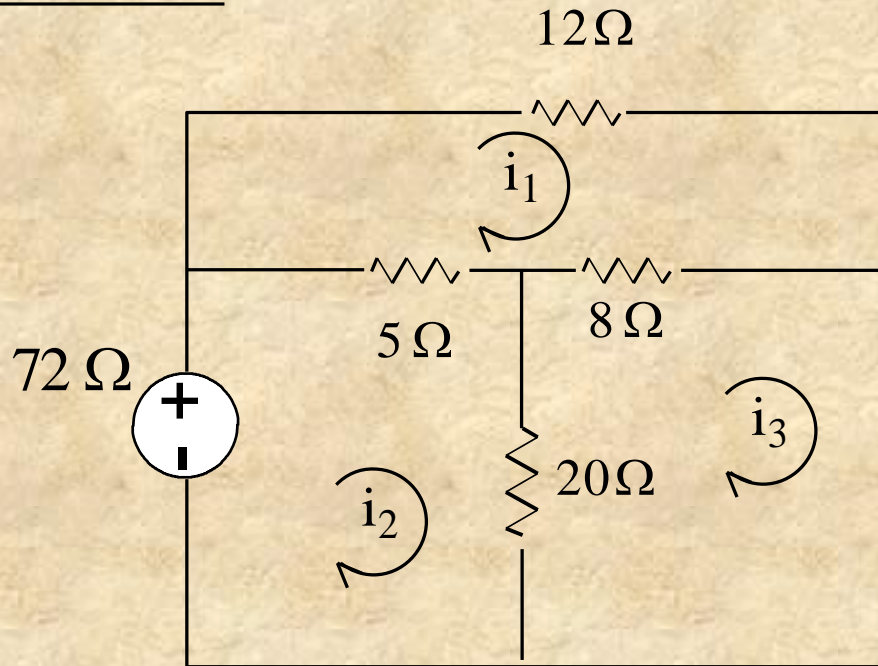
$$V_o = \frac{4}{4 + R_{TH}} V_{oc}$$

$$= \frac{4}{4 + 6} (64.8)$$

$$V_o = 25.92 \text{ V}$$

## Using Norton Theorem:

1. Find  $I_{SC}$ :



KVL around upper loop :

$$12 i_1 + 8 (i_1 - i_3) + 5 (i_1 - i_2) = 0$$

$$25 i_1 - 5 i_2 - 8 i_3 = 0 \quad \dots\dots(1)$$

KVL around lower loop :

$$\begin{aligned}5 (i_2 - i_1) + 20 (i_2 - i_3) &= 72 \\ -5 i_1 + 25 i_2 - 20 i_3 &= 72 \quad \dots\dots(2)\end{aligned}$$

KVL around right loop :

$$\begin{aligned}8 (i_3 - i_1) + 20 (i_3 - i_2) &= 0 \\ -8 i_1 - 20 i_2 + 28 i_3 &= 0 \quad \dots\dots(3)\end{aligned}$$

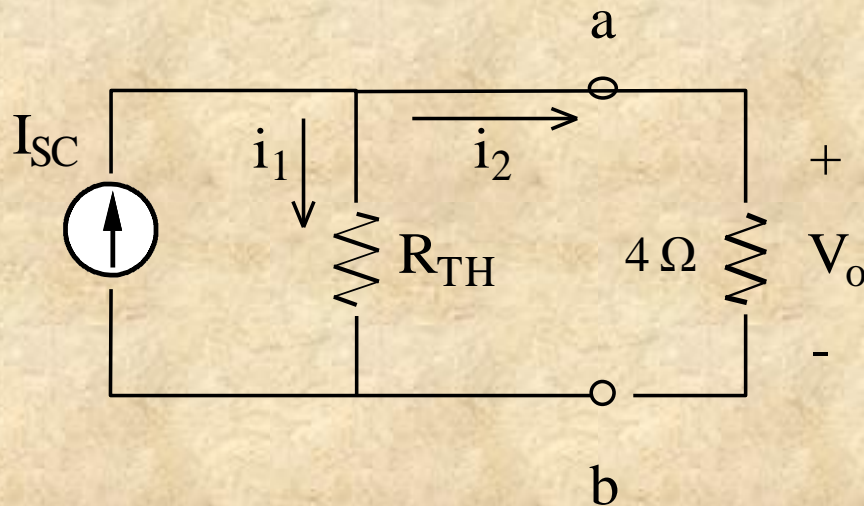
$$i_1 = 6 \text{ A}, \quad i_2 = 12.72 \text{ A}, \quad i_3 = 10.8 \text{ A}$$

$$\Rightarrow I_{SC} = 10.8 \text{ A}$$

## 2. Find $R_{TH}$

From before ,  $R_{TH} = 6 \Omega$

## 3. Reconnect the load



$$V_o = (4 \Omega) i_2$$

$$= (4 \Omega) \left( \frac{R_{TH}}{R_{TH} + 4} \right) I_{SC}$$

$$V_o = 4 \left( \frac{6}{6+4} \right) (10.8) = 25.92 \text{ V}$$



## Case(2) : Circuits containing only dependent sources

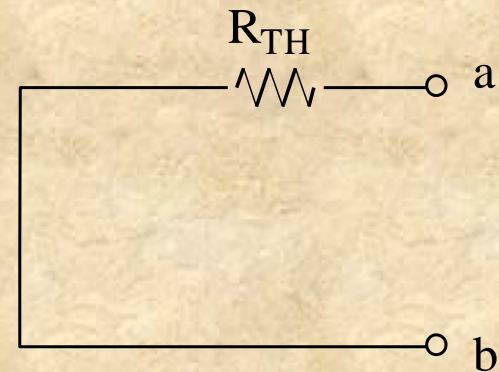
Here there is NO energy source in the circuit.

➤  $V_{OC}$  is always zero and  $I_{SC}$  is always zero

➤ So we can only find  $R_{TH}$

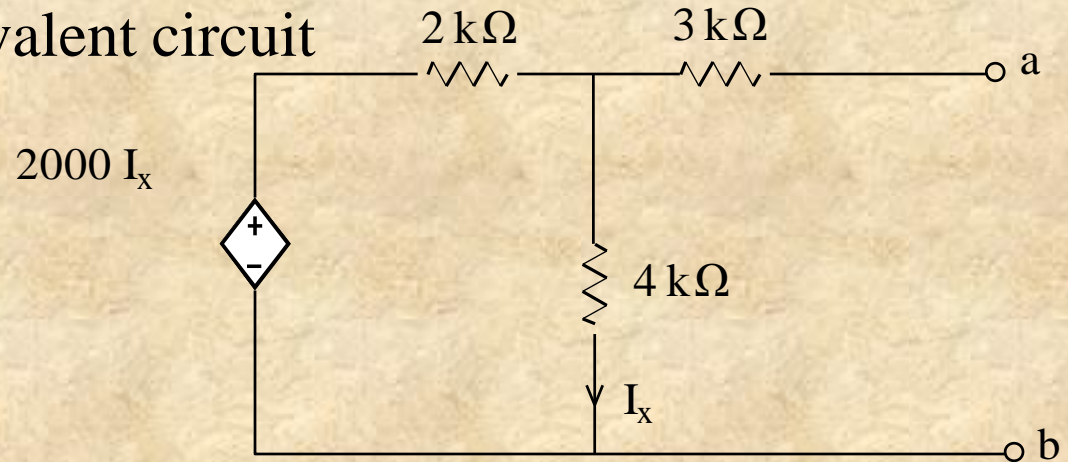
### Procedure for finding $R_{TH}$

1. Connect an independent voltage ( or current) source at the terminals ,  $V_X$  (or  $I_X$ )
2. Find the corresponding current ( or voltage) at the terminal ,  $I_o$  ( or  $V_o$ )
3. Find  $R_{TH} = V_X / I_o$  or  $R_{TH} = V_o / I_X$

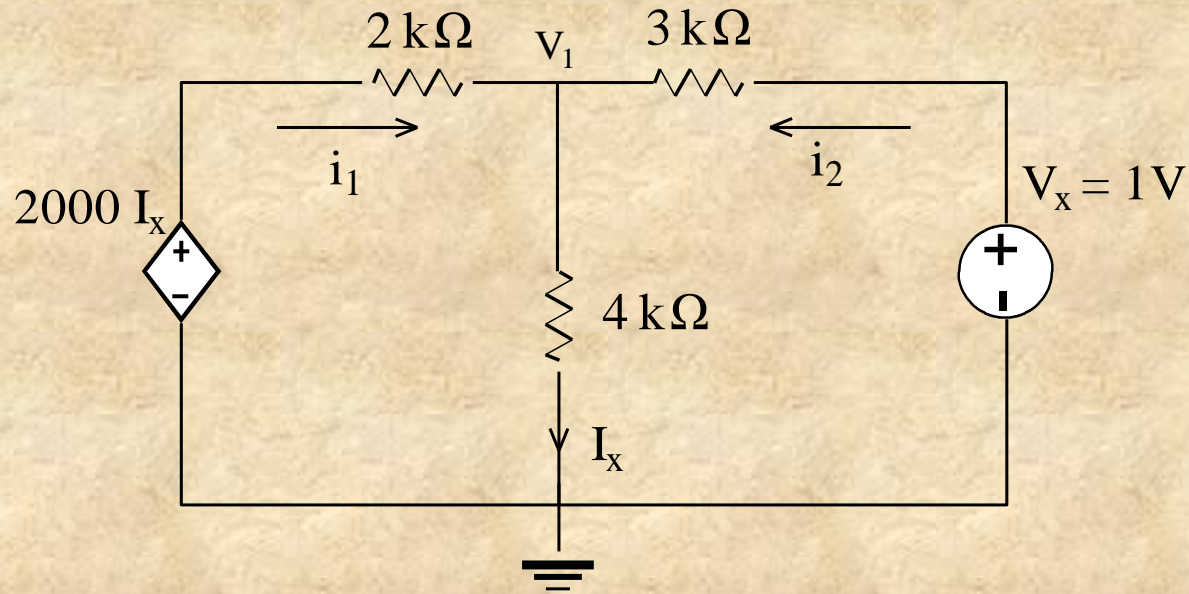


## Example:

Find the Thevenin equivalent circuit



1. Apply voltage source at the terminals ( $V_x = 1\text{ V}$ )



KCL at node V1 :

$$i_1 + i_2 - I_X = 0$$

$$\frac{2000 I_X - V_1}{2 \text{ k}} + \frac{1 - V_1}{3 \text{ k}} - I_X = 0$$

where  $V_1 = (4 \text{ k}) I_X$

$$\frac{2000 I_X - 4000 I_X}{2000} + \frac{1 - 4000 I_X}{3000} - I_X = 0$$

$$I_X - 2 I_X + \frac{1}{3 \text{ k}} - \frac{4}{3 \text{ k}} I_X - I_X = 0$$

$$I_X \left[ 2 + \frac{4}{3} \right] = \frac{1}{3000}$$

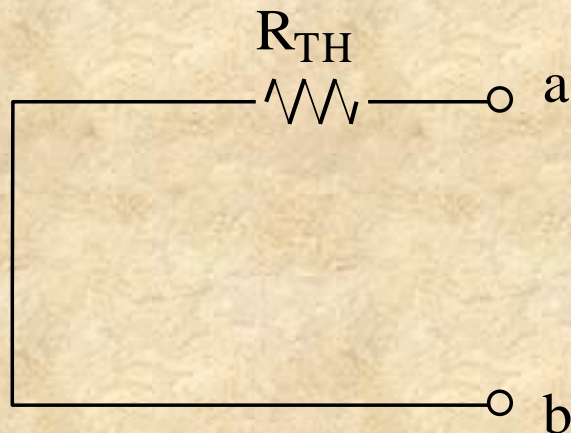
$$I_X = 0.1 \text{ m A}$$

$$i_2 = \frac{V_X - V_1}{3 \text{ k}}$$

$$= \frac{V_X - (4 \text{ k}) I_X}{3 \text{ k}} = \frac{1 - (4 \text{ k})(0.1 \text{ m A})}{3 \text{ k}}$$

$$i_2 = 0.2 \text{ m A}$$

$$R_{\text{TH}} = \frac{V_X}{i_2} = \frac{1 \text{ V}}{0.2 \text{ m A}} = 5 \text{ k } \Omega$$



## Case (3) : Circuits containing both independent and dependent sources

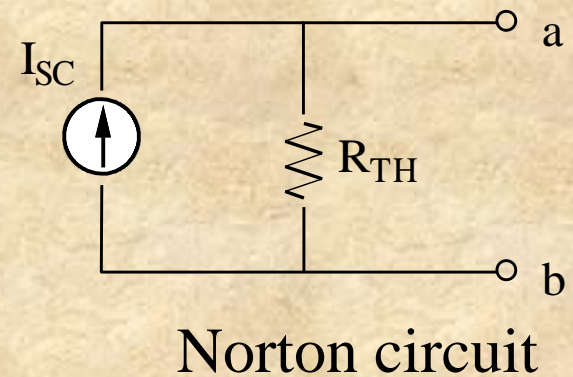
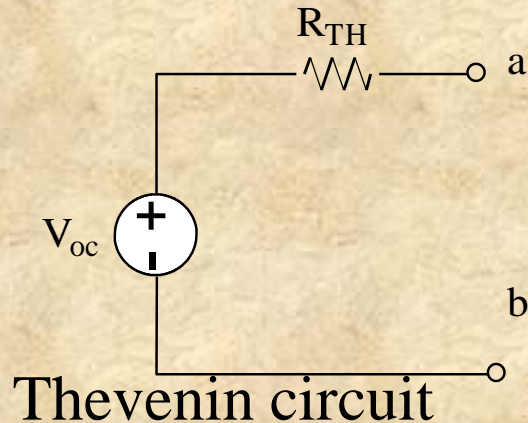
### Procedure of Thevenin or Norton Theorms:

- Find the open circuit voltage and the terminals,  $V_{OC}$
- Find the short circuit current at the terminals,  $I_{SC}$ .
- Compute  $R_{TH} = V_{OC}/I_{SC}$

### Note :

$R_{TH}$  can not be found as in the case of only independent sources

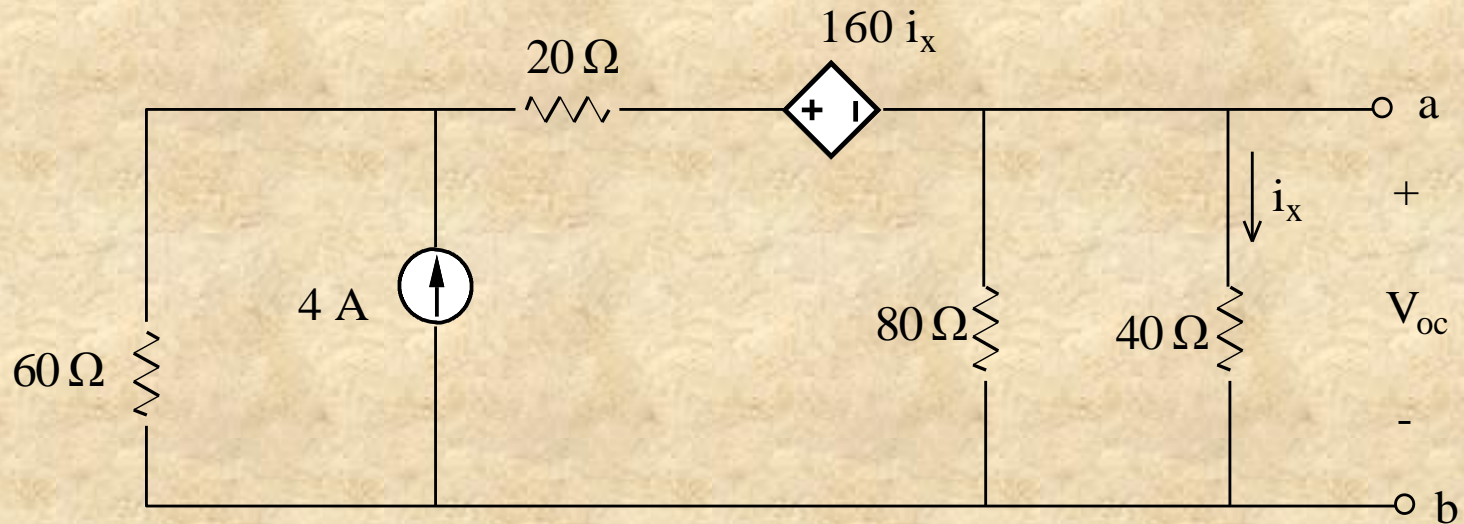
- Construct the Thevenin or Norton circuits



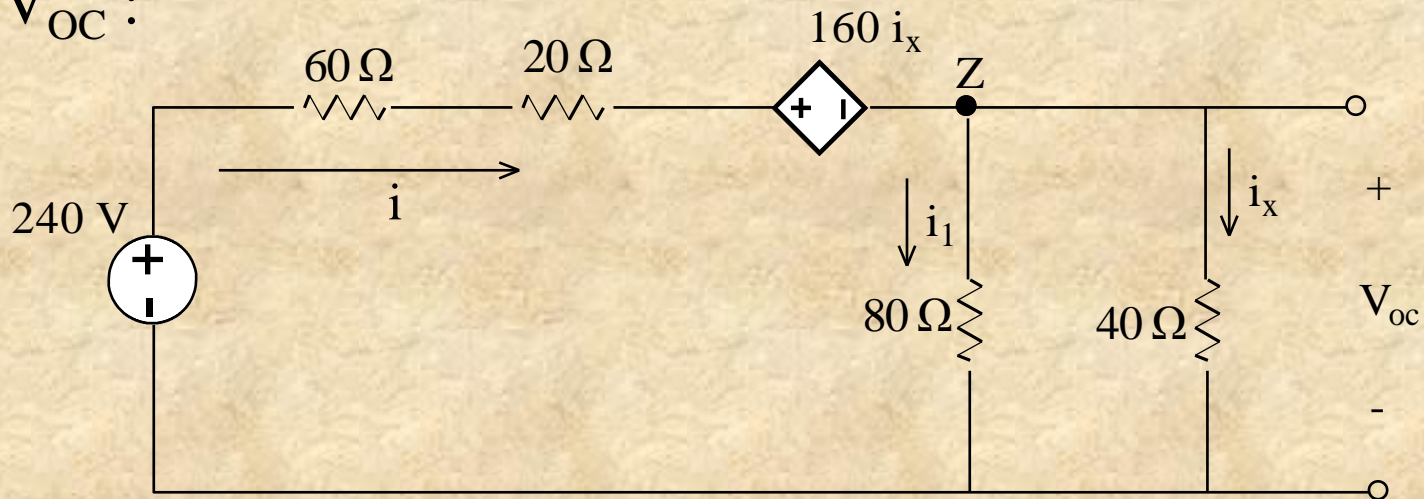


## Example :

Find the Thevenin equivalent circuit with respect to the terminals a, b



1. Find  $V_{oc}$  :



KVL around the left loop :

$$-240 + 80 i + 160 i_x + 40 i_x = 0$$

$$80 i + 200 i_x = 240 \quad \dots\dots(1)$$

KVL around right loop :

$$80 i_1 = 40 i_x$$

$$2 i_1 - i_x = 0 \quad \dots\dots(2)$$

KCL at Z:

$$i - i_1 - i_x = 0 \quad \dots\dots(3)$$

$$i = 1.125 \text{ A}$$

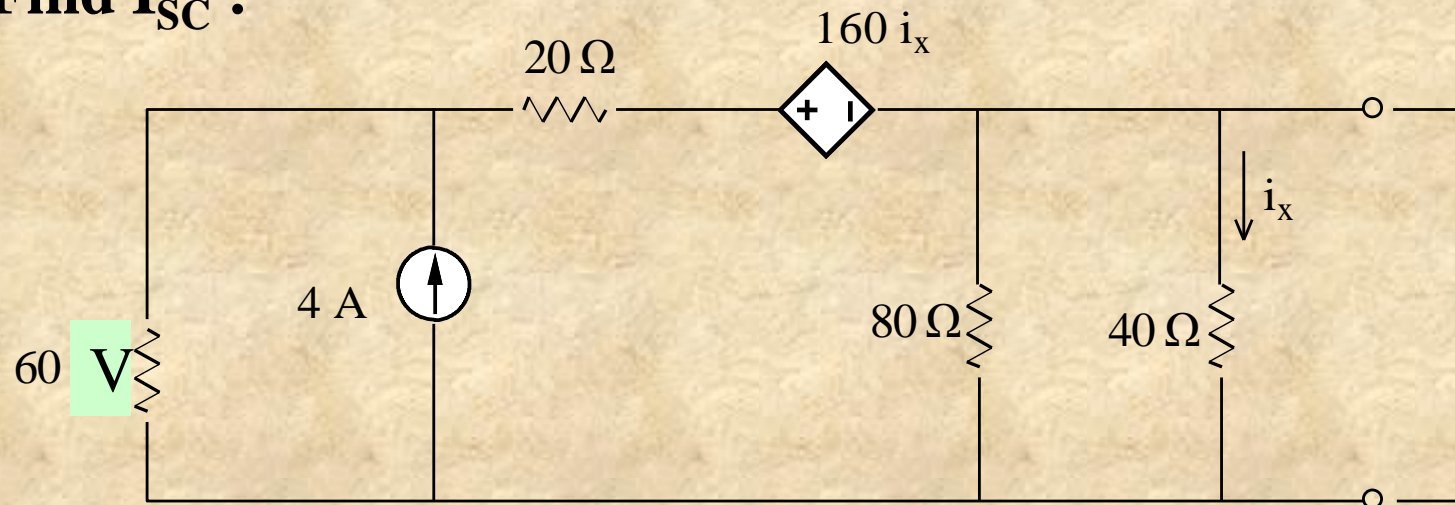
$$i_1 = 0.375 \text{ A}$$

$$i_x = 0.75 \text{ A}$$

$$\begin{aligned} \therefore V_{OC} &= i_x (40 \Omega) \\ &= (0.75 \text{ A}) (40 \Omega) \end{aligned}$$

$$V_{OC} = 30 \text{ V}$$

**2. Find  $I_{SC}$  :**



Since we have short circuit ,  $80 \parallel 40 \parallel 0 = 0$

⇒  $i_x = 0$

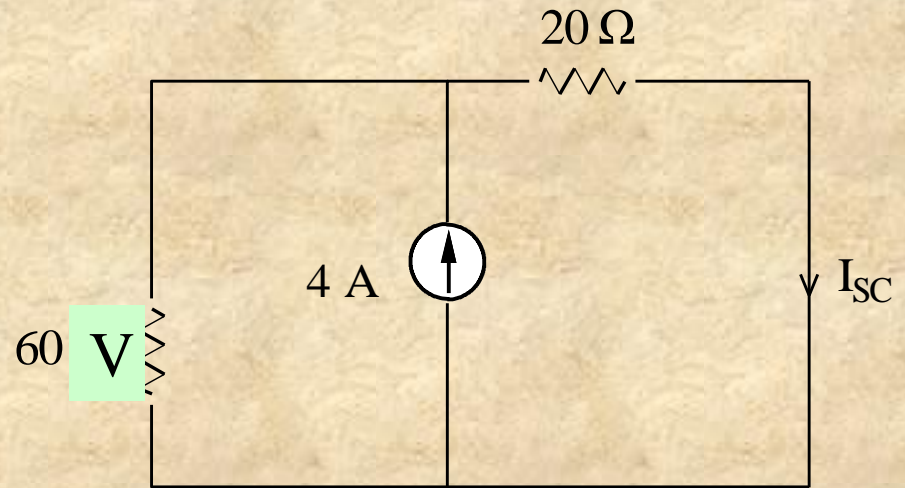
⇒  $160i_x$  source is zero

$$I_{SC} = \frac{60}{60 + 20} (4) = 3 \text{ A}$$

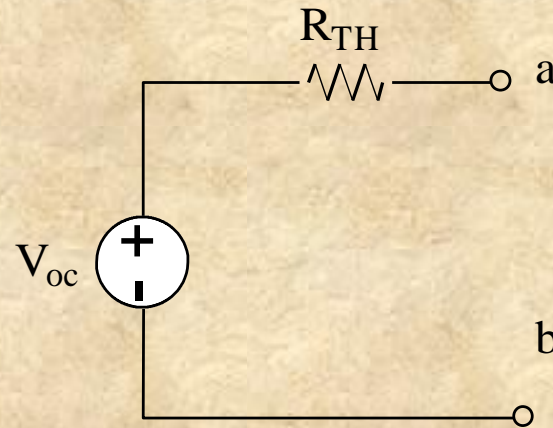
3. Find  $R_{TH}$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{30 \text{ V}}{3 \text{ A}} = 10 \Omega$$

4. ⇒

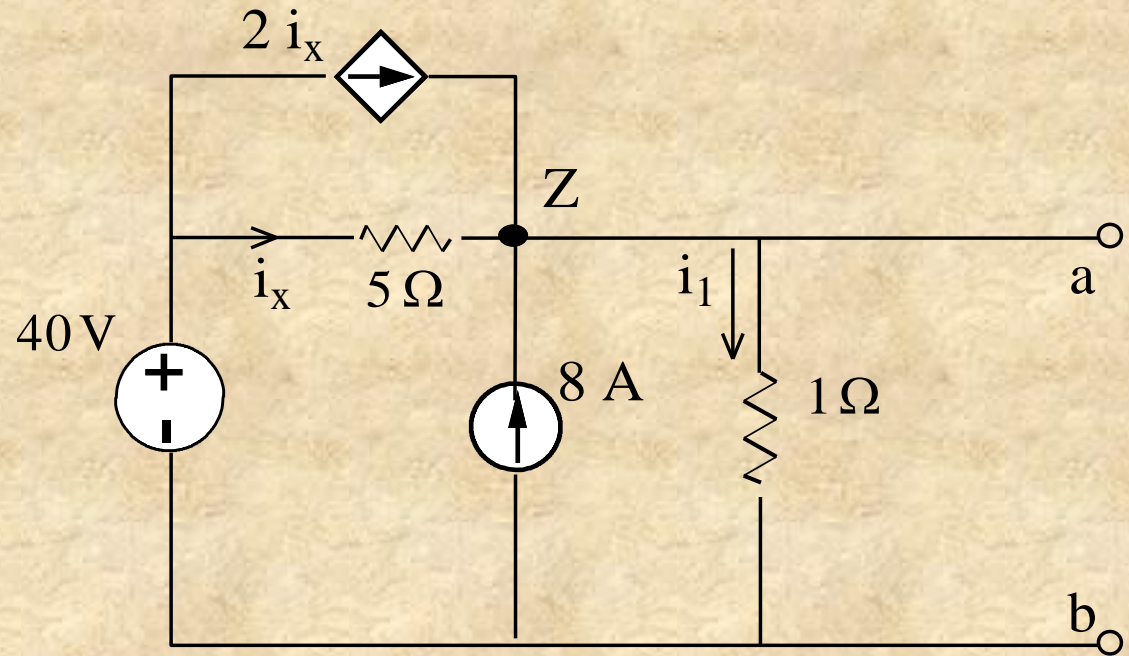


Current divider



## Example :

Use Thevenin theorem to find the Thevenin equivalent circuit with respect to a, b



1. Find  $V_{OC}$

KCL at node z :

$$2i_x + i_x + 8 - i_1 = 0$$

$$3i_x - i_1 = -8 \quad \dots\dots(1)$$



KVL around outer loop

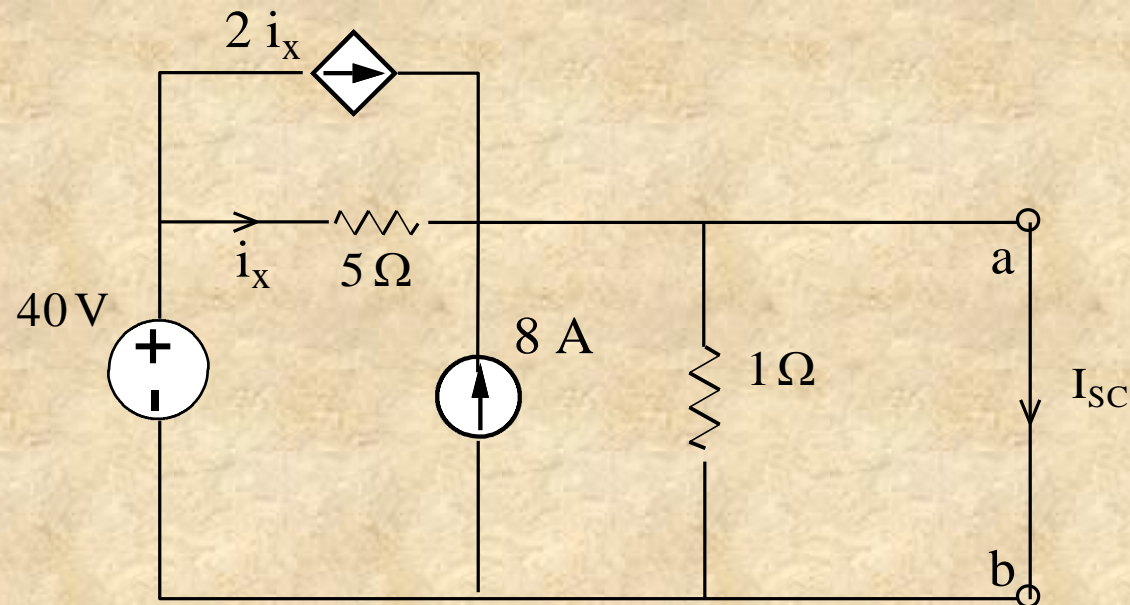
$$-40 + 5 i_x + 1 i_1 = 0$$

$$5 i_x + i_1 = 40 \quad \dots\dots(2)$$

$$\Rightarrow i_x = 4 \text{ A} \quad , \quad i_1 = 20 \text{ A}$$

$$\Rightarrow V_{OC} = 1 i_1 = 20 \text{ V}$$

Find  $I_{SC}$  :



KVL around outer loop :

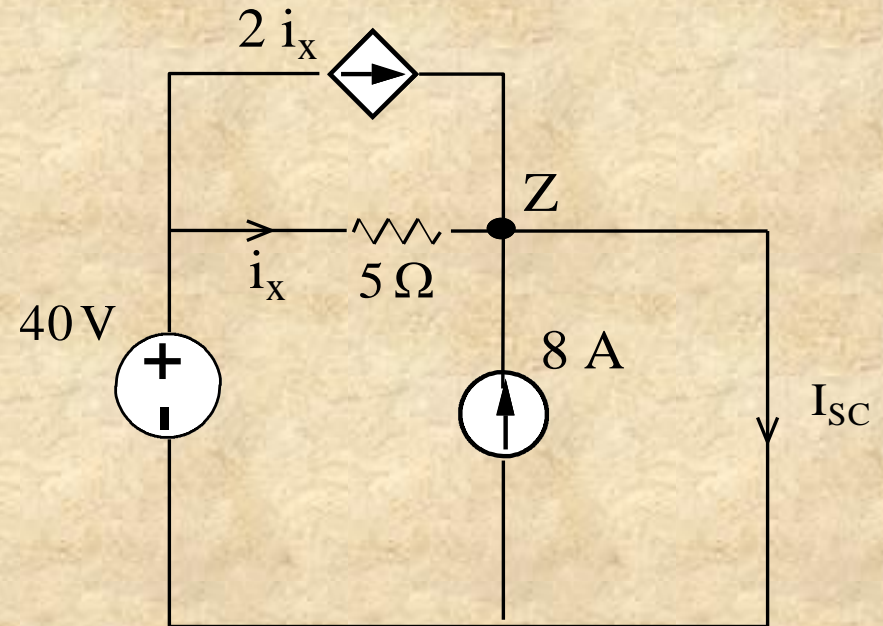
$$-40 + 5 i_x = 0 \Rightarrow i_x = 8 \text{ A}$$

KCL at z :

$$2 i_x + i_x + 8 = I_{SC}$$

$$3 i_x + 8 = I_{SC}$$

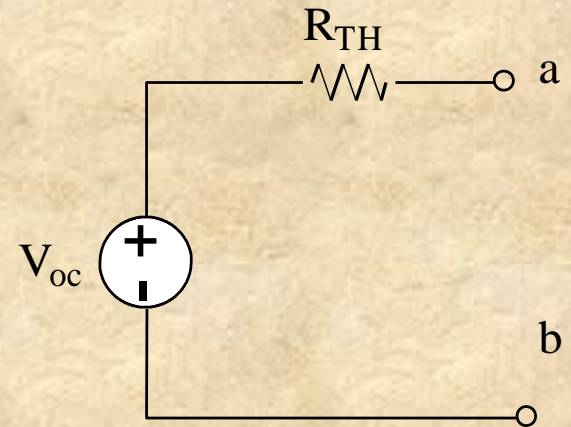
$$\Rightarrow I_{SC} = (3)(8) + 8 = 32 \text{ A}$$



3. Find  $R_{TH}$  :

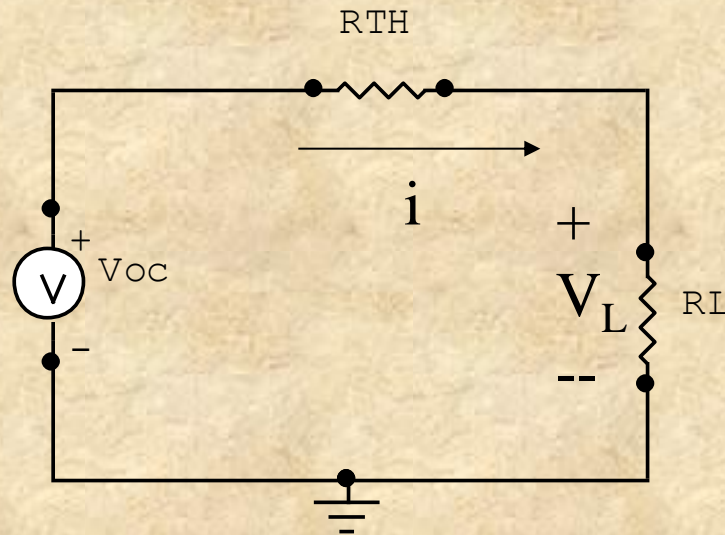
$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{20}{32} = 0.625 \Omega$$

Thevenin equivalent circuit is



## 4. Maximum Power Transfer

- A technique in which the load is selected to maximize the power transfer.
- This technique is based on the Thevenin equivalent circuit.



$$P_L = V_L i = i^2 R_L$$
$$= \left( \frac{V_{OC}}{R_{TH} + R_L} \right)^2 R_L$$

We wish to select  $R_L$  to maximize  $P_L$ :

$$\text{Take } \frac{dP_L}{dR_L} = 0$$

$$\frac{dP_L}{dR_L} = \frac{(R_{TH} + R_L)^2 (V_{OC})^2 - R_L (V_{OC})^2 \cdot 2(R_{TH} + R_L)}{(R_{TH} + R_L)^4} = 0$$

$$\frac{V_{OC}^2 (R_{TH} + R_L) [(R_{TH} + R_L) - 2 R_L]}{(R_{TH} + R_L)^4} = 0$$

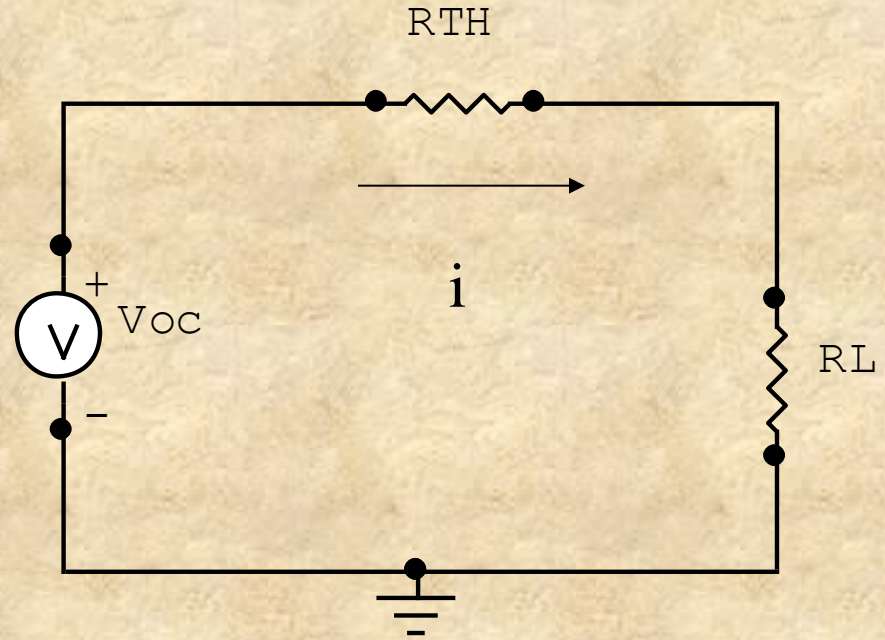
$$\Rightarrow R_{TH} + R_L - 2 R_L = 0$$

$$\Rightarrow R_{TH} - R_L = 0$$

$$R_L = R_{TH}$$

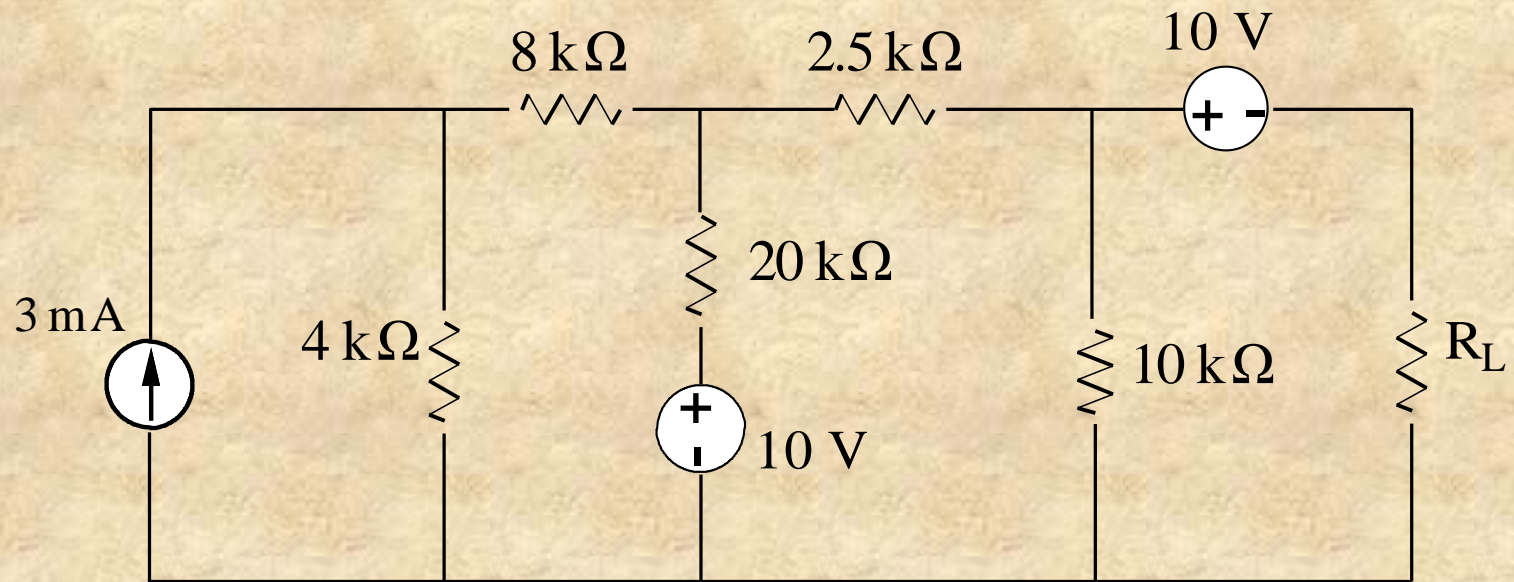
If  $R_L = R_{TH}$ , what is the maximum Power Transfer?

$$\begin{aligned}
 P_{L_{\max}} &= i^2 R_L \\
 &= \left( \frac{V_{OC}}{2 R_{TH}} \right)^2 R_{TH} \\
 &= \frac{(V_{OC})^2 R_{TH}}{4 R_{TH}^2} = \frac{(V_{OC})^2}{4 R_{TH}} \\
 P_{L_{\max}} &= \frac{V_{OC}^2}{4 R_{TH}}
 \end{aligned}$$



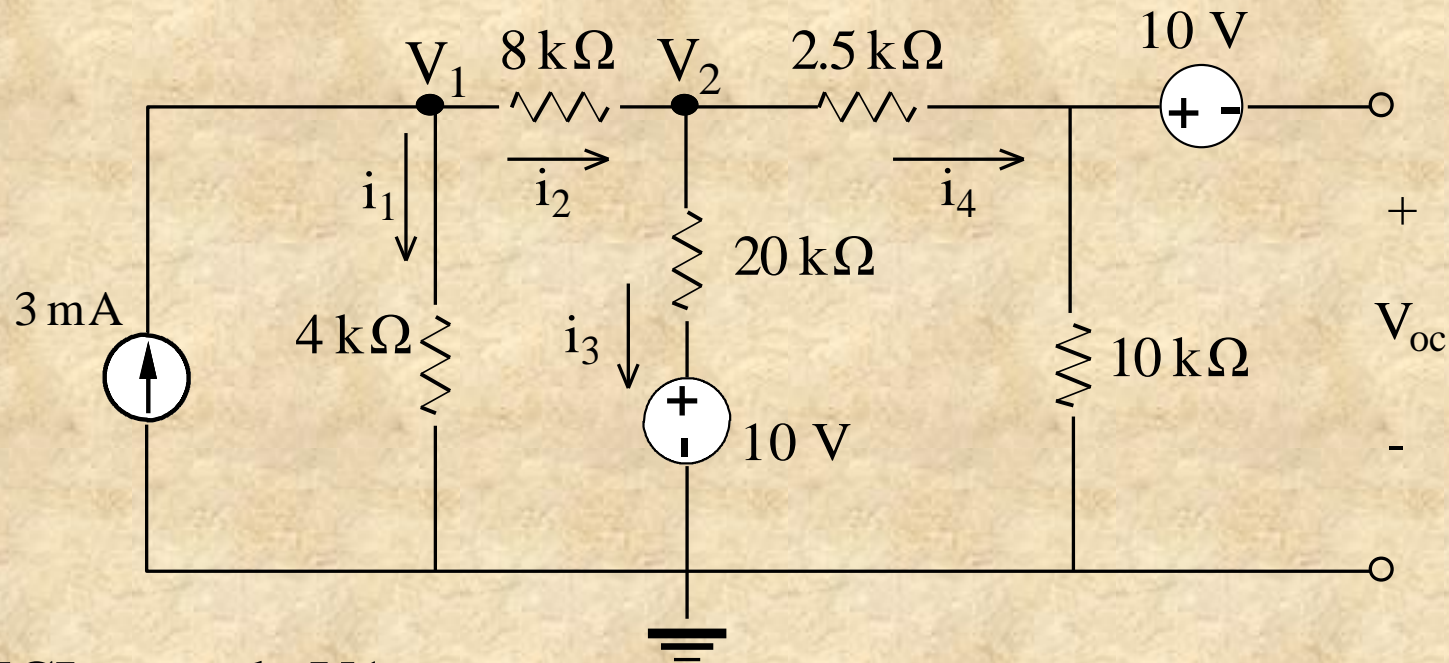


## Example:



- Find  $R_L$  for maximum Power Transfer ?
- Find the maximum Power transfer to  $R_L$  ?

Let's find Thevenin equivalent circuit .



KCL at node V1 :

$$3 \text{ m A} - i_1 - i_2 = 0$$

$$3 \text{ m A} - \frac{V_1}{4 \text{ k } \Omega} - \frac{V_1 - V_2}{8 \text{ k } \Omega} = 0$$

$$V_1 \left[ \frac{1}{4 \text{ k}} + \frac{1}{8 \text{ k}} \right] - \left( \frac{1}{8 \text{ k}} \right) V_2 = 3 \text{ m}$$

$$0.375 \text{ m } V_1 - 0.125 \text{ m } V_2 = 3 \text{ m} \quad \dots\dots(1)$$

KCL at node V2:

$$i_2 - i_3 - i_4 = 0$$

$$\frac{V_1 - V_2}{8 \text{ k}} - \frac{V_2 - 10}{20 \text{ k}} - \frac{V_2}{12.5 \text{ k}} = 0$$

$$V_1 \left( \frac{1}{8 \text{ k}} \right) - \left( \frac{1}{8 \text{ k}} + \frac{1}{20 \text{ k}} + \frac{1}{12.5 \text{ k}} \right) V_2 = -0.5 \text{ m}$$

$$0.125 \text{ m } V_1 - 0.255 \text{ m } V_2 = -0.5 \text{ m} \quad \dots\dots(2)$$

$$V_1 = 10.34 \text{ V}$$

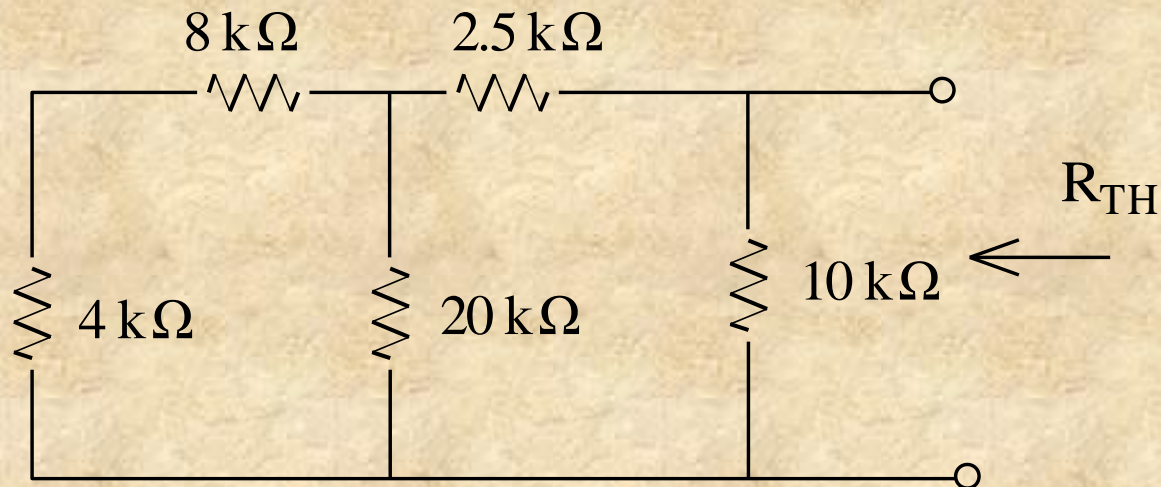
$$V_2 = 7.03 \text{ V}$$

$$V_{OC} = -10 + 10 \text{ k } i_4$$

$$= -10 + 10 \text{ k } \left( \frac{V_2}{12.5 \text{ k}} \right) = -10 + \frac{10}{12.5} (7.03)$$

$$V_{OC} = -4.375 \text{ V}$$

To find  $R_{TH}$ :



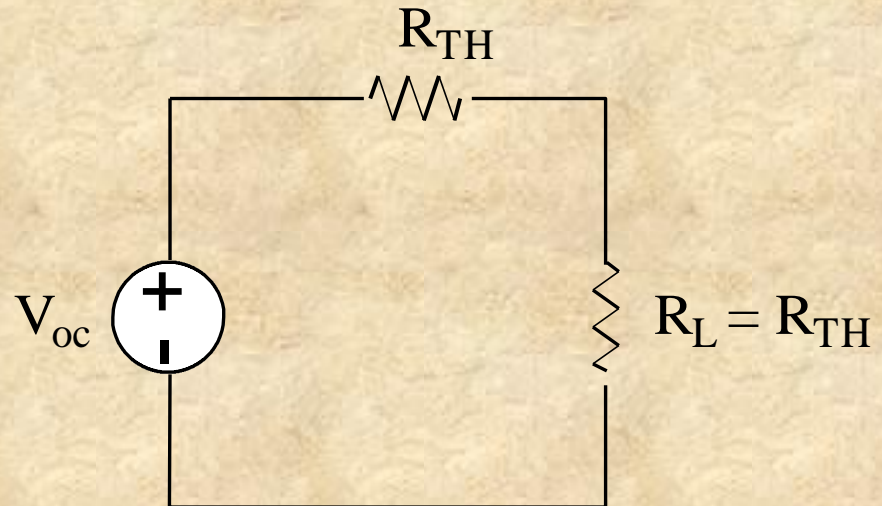
$$R_{TH} = \{ [(8 \text{ k} + 4 \text{ k}) // 20 \text{ k}] + 2.5 \text{ k} \} // 10 \text{ k}$$

$$= [(12 \text{ k} // 20 \text{ k}) + 2.5 \text{ k}] // 10 \text{ k}$$

$$= (7.5 \text{ k} + 2.5 \text{ k}) // 10 \text{ k}$$

$$= 10 \text{ k} // 10 \text{ k}$$

$$R_{TH} = 5 \text{ k } \Omega$$

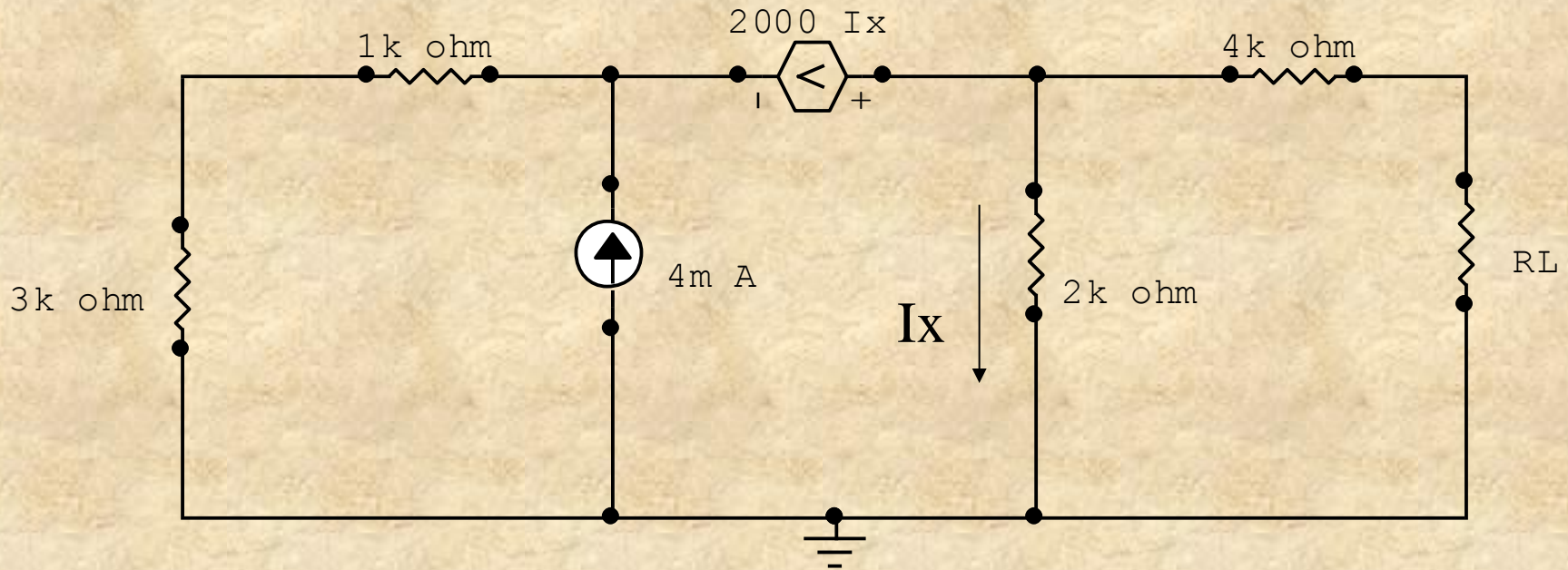


$$P_{L_{max}} = \frac{V_{OC}^2}{4 R_{TH}} = \frac{(-4.375)^2}{4 (5 \text{ k})}$$

$$P_{L_{max}} = 0.957 \text{ m W}$$

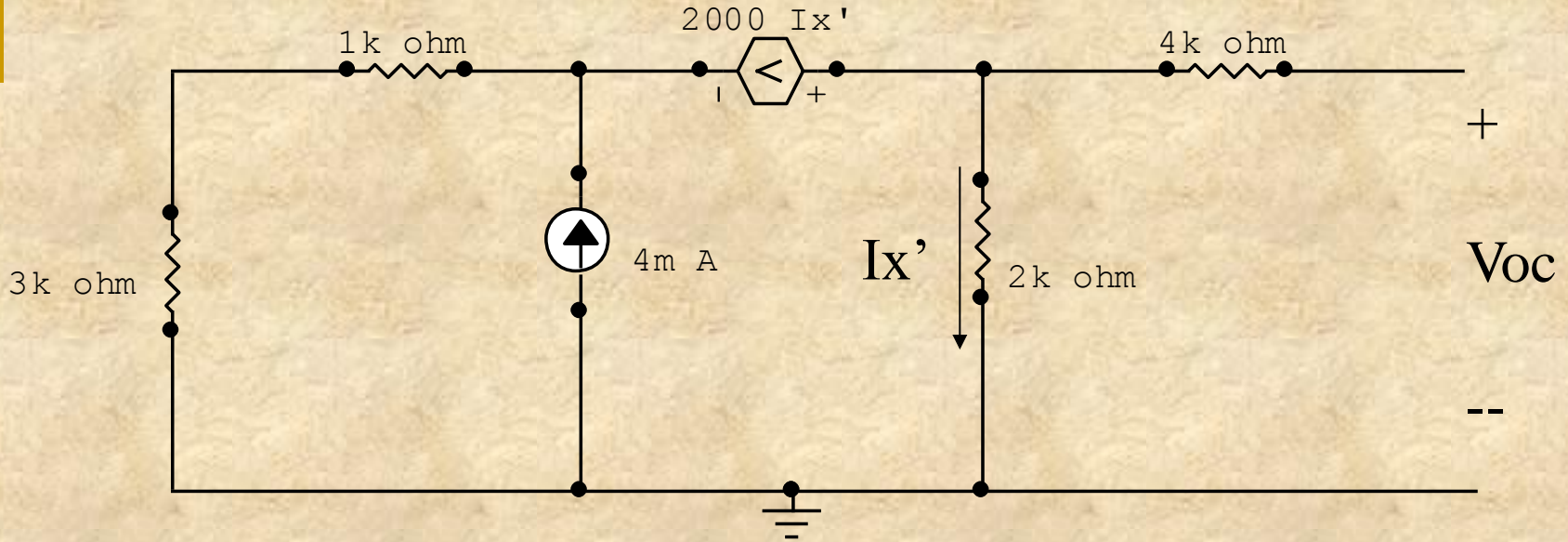


## Example :

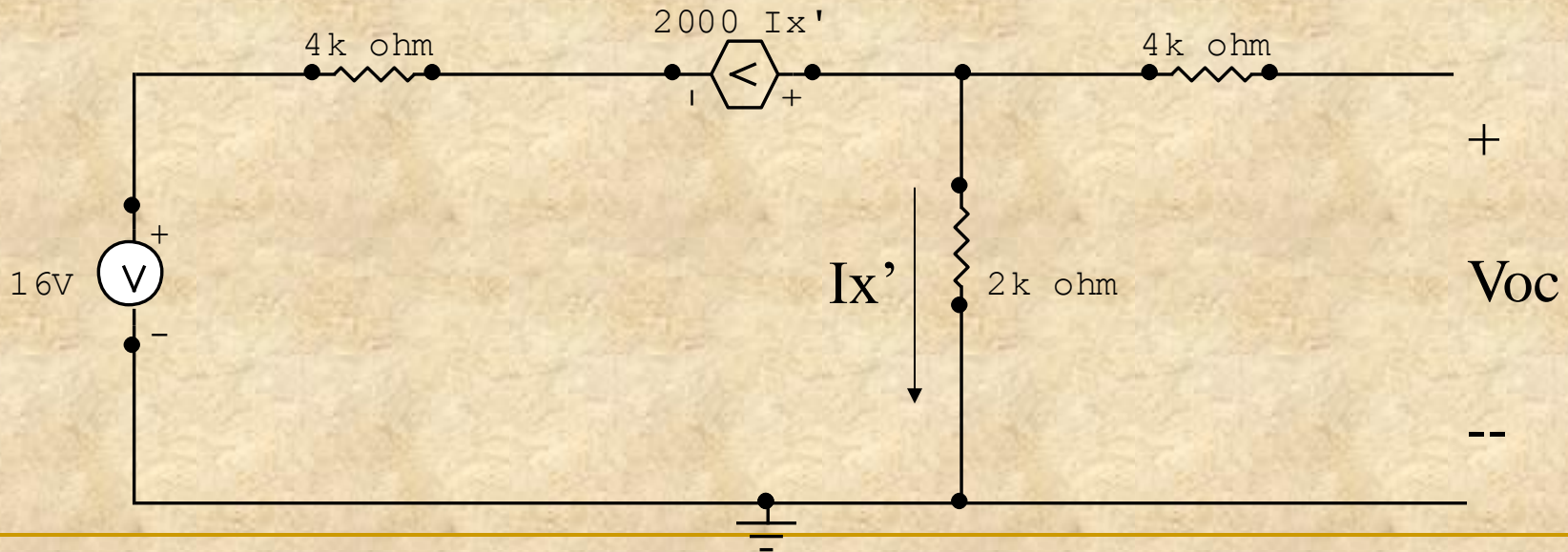


1. Find  $R_L$  for maximum Power Transfer?
2. Find max. power transfer to  $R_L$  ?

First , find Thevenin equivalent:



Using source transformation



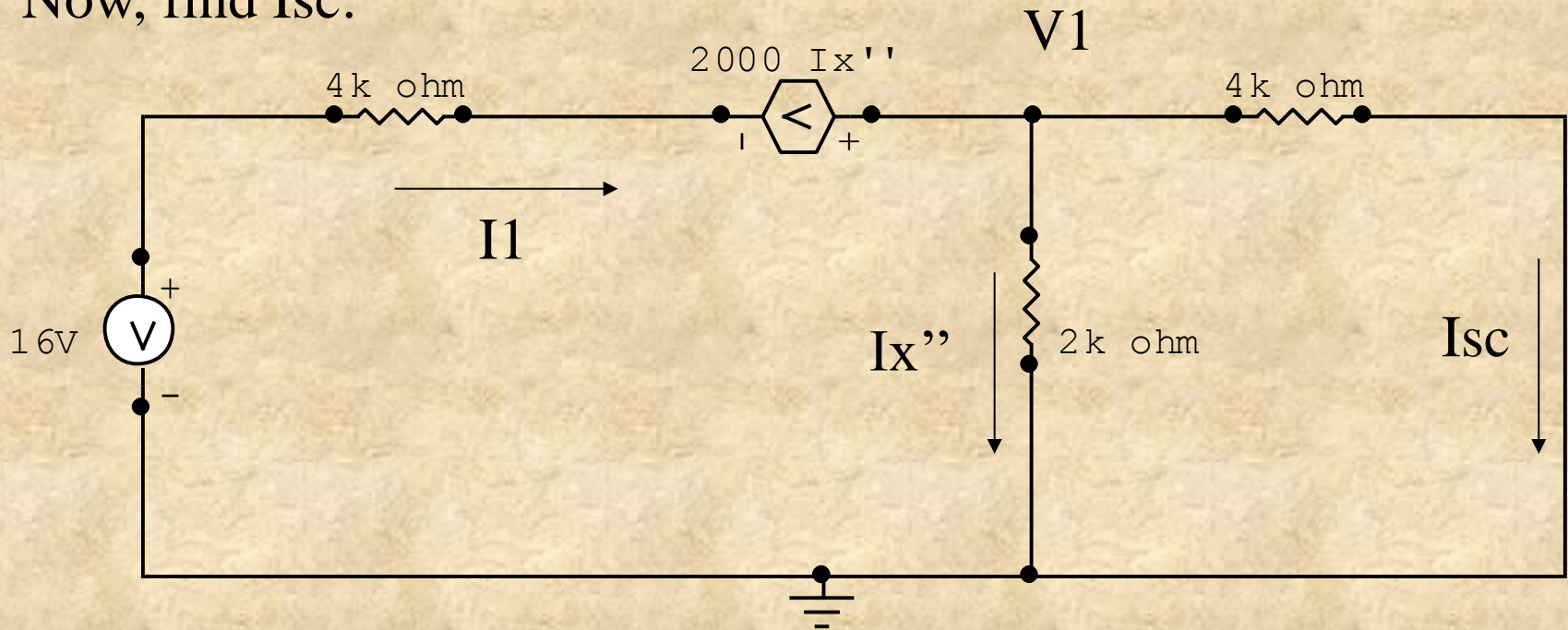
KVL around the loop:

$$-16 + 4k I_x' - 2k I_x' + 2k I_x' = 0$$

$$I_x' = 4\text{mA}$$

$$V_{oc} = (2k\Omega) I_x' = 8\text{V}$$

Now, find  $I_{sc}$ :



KCL at V1:

$$I_1 - I_{x''} - I_{sc} = 0$$

$$\frac{16 - (V_1 - 2kI_{x''})}{4k} - \frac{V_1}{2k} - \frac{V_1}{4k} = 0$$

Where  $V_1 = 2k I_{x''}$

Hence,

$$\frac{16}{4k} - \frac{V_1}{2k} - \frac{V_1}{4k} = 0 \quad \text{Or } V_1 = 5.333V$$

And

$$I_{sc} = \frac{V_1}{4k} = 1.333mA$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{8\text{ V}}{1.333\text{ mA}} = 6\text{ k}\Omega$$

$$P_{L(\max)} = \frac{V_{OC}^2}{4R_{TH}} = \frac{(8)^2}{4(6\text{ k})} = \frac{64}{24\text{ k}}$$

$$P_{L(\max)} = \frac{8}{3}\text{ m W}$$