Required text

*College Physics*
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BROOKS/COLE CENGAGE Learning
Course Outline

Chapter 6

**Momentum and Collisions**

6.1 Momentum and Impulse
6.2 Conservation of Momentum
6.3 Collisions
6.4 Glancing Collisions
Chapter 6

Momentum and Collisions

6.1 Momentum and Impulse
6.2 Conservation of Momentum
6.3 Collisions
6.4 Glancing Collisions
6.1 Momentum and Impulse

◆ **Linear momentum** – the linear momentum of an object of mass \( m \) moving at speed \( v \) is expressed as

\[
\vec{p} = m\vec{v}
\]

◆ SI unit of momentum \( \rightarrow \) kg m/s

◆ Newton used the concept of momentum to express his second law:

\[
\vec{F} = m\vec{a} = \frac{m\Delta \vec{v}}{\Delta t}
\]

\[
\vec{F} = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right)
\]

\[
\vec{F} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t}
\]
6.1 Momentum and Impulse

- Force is also a measurement of change in linear momentum over a given time interval.
  \[ \vec{F} = \vec{p}_f - \vec{p}_i = \frac{\Delta \vec{p}}{\Delta t} \]

- For an object in equilibrium
  \[ \vec{F}_{\text{ext}} = 0 \Rightarrow \Delta p = 0 \Rightarrow p_i = p_f \]

=> If the sum of all of the external forces equals zero, momentum remains constant and is therefore conserved.
6.1 Momentum and Impulse

◆ **Impulse** – In the initial seconds of a collision, there is an *impulse force* on the object.
◆ This force is defined as the change in linear momentum:

\[ \vec{I} = \vec{F} \Delta t = \Delta \vec{p} = m \vec{v}_f - m \vec{v}_i \]

◆ In order to **change** the momentum of an object, a force must be applied.
◆ The time rate of change of momentum of an object is equal to the net force acting on it

\[ \frac{\Delta \vec{p}}{\Delta t} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t} = \vec{F}_{\text{net}} \]
6.1 Momentum and Impulse

- It gives an alternative statement of Newton’s second law

\[ \vec{I} = \vec{F} \Delta t \]

- Impulse is a vector quantity, the direction is the same as the direction of the force.

**Impulse-Momentum Theorem**

- The theorem states that the impulse acting on the object is equal to the change in momentum of the object

\[ \vec{I} = \vec{F} \Delta t = \Delta \vec{p} = m \vec{v}_f - m \vec{v}_i \]
6.1 Momentum and Impulse

◆ If the force is not constant, use the average force applied.

**Average Force in Impulse**

◆ The average force can be thought of as the constant force that would give the same impulse to the object in the time interval as the actual time-varying force gives in the interval:
6.1 Momentum and Impulse

Average Force in Impulse

◆ The impulse imparted by a force during the time interval $\Delta t$ is equal to the area under the force-time graph from the beginning to the end of the time interval.

◆ Or, the impulse is equal to the average force multiplied by the time interval,

$$\overrightarrow{F}_{\text{av}} \Delta t = \Delta \vec{p}$$
6.1 Momentum and Impulse

Impulse Applied to Auto Collisions

◆ The most important factor is the collision time or the time it takes the person to come to a rest. E.g., this will reduce the chance of dying in a car crash.

◆ Ways to increase the time:
  ✔ Seat belts
  ✔ Air bags
6.1 Momentum and Impulse

Acceleration That Would Kill a Human

- When placing a human under high acceleration the brain, lungs, ribs (any bones), etc., all affect the death of the human.
- Since different body parts have different densities, certain organs will undergo more g-force than others.
- Pilots, roller coaster rides, and car crashes are sufficient examples of g-force.
- Pilots are trained to undergo accelerations of 9 g's to pull quick maneuvers during flight for less than a second.
- If a force of 4 to 6 g's is held for more than a few seconds, the results could be devastating.
6.1 Momentum and Impulse

Typical Collision Values

- For a 75 kg person traveling at 27 m/s and coming to stop in 0.010 s
- \( F = -2.0 \times 10^5 \) N
- \( a = 280 \text{ g} \): Almost certainly fatal
- The 1997 car crash that killed Princess Diana was estimated to range somewhere between 70–100 g's.
- In general, high velocity doesn't kill; what kills is the high acceleration or deceleration sustained over a certain time interval.
6.1 Momentum and Impulse

Survival secret: Increase time

🔹 Seat belt: Restrain people so it takes more time for them to stop: about 0.15 seconds
🔹 Air Bag:
  ✔ It increases the time of the collision
  ✔ It will also absorb some of the energy from the body
  ✔ It will spread out the area of contact (decreases the pressure, it helps prevent penetration wounds)
An estimated force vs. time curve for a baseball struck by a bat is shown in Figure. From this curve, determine:
(a) the impulse delivered to the ball and
(b) the average force exerted on the ball.
Problems
◆ Problem 6.1

Solution
(a) The impulse imparted by the force during the time interval $\Delta t$ is equal to the area under the force-time graph from the beginning to the end of the time interval:

$$\Delta \vec{p} = \bar{F}_{av} \Delta t = \frac{1.5{(ms)} \cdot 18000{(N)}}{2} = \frac{1.5 \cdot 18{(N \cdot s)}}{2} = 13.5{(N \cdot s)}$$

(b) The average force:

$$\bar{F}_{av} = \frac{\Delta \vec{p}}{\Delta t} = \frac{13.5{(N \cdot s)}}{1.5{ms}} = 9000{(N)}$$
Chapter 6

Momentum and Collisions

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6.2 Conservation of Momentum

- When two isolated objects collide, the impulse exerted by \( m_2 \) on \( m_1 \) is
  \[ \Delta t \vec{F}_1 = m_1 \vec{v}_{1f} - m_1 \vec{v}_{1i} \]
  while the impulse exerted by \( m_1 \) on \( m_2 \) is
  \[ \Delta t \vec{F}_2 = m_2 \vec{v}_{2f} - m_2 \vec{v}_{2i} \]
- According to Newton’s third law, \( \vec{F}_1 = -\vec{F}_2 \)
- Hence, combining above Eq. we prove that momentum is conserved during the collision:
6.2 Conservation of Momentum

Momentum is conserved during the collision:

\[ \Delta t \left( \vec{F}_1 + \vec{F}_2 \right) = m_1 \vec{v}_{1f} - m_1 \vec{v}_{1i} + m_2 \vec{v}_{2f} - m_2 \vec{v}_{2i} \]

\[ m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} - \left( m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} \right) = 0 \]

\[ \vec{p}_f - \vec{p}_i = 0 \]

\[ \vec{p}_f = \vec{p}_i \]

The principle of conservation of momentum states: when no external forces act on a system consisting of two objects that collide with each other, the total momentum of the system remains constant in time.
6.2 Conservation of Momentum

- More precisely, the total momentum before the collision will equal the total momentum after the collision.
- The momentum of each object will change.
- The total momentum of the system remains constant.
6.2 Conservation of Momentum

Forces in a collision

- The force with which object 1 acts on object 2 is equal and opposite to the force with which object 2 acts on object 1.
- Impulses are also equal and opposite.
6.2 Conservation of Momentum

- Mathematically: \[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]
- Momentum is conserved for the system of objects.
- The system includes all the objects interacting with each other.
- Assumes only internal forces are acting during the collision.
- Can be generalized to any number of objects.
Problems
◆ Problem 6.2

A rifle with a weight of 30 N fires a 5.0-g bullet with a speed of 300 m/s.

Questions:
(a) Find the recoil speed of the rifle.
(b) If a 700-N man holds the rifle firmly against his shoulder, find the recoil speed of man and rifle.
Problems

Problem 6.2

Solution

(a) Choosing the direction of the bullet's motion as positive,

\[ m_R v_R + m_b v_b = 0, \]

which gives

\[ v_R = -v_b \left( \frac{m_b}{m_R} \right) \]

\[ = -300 \text{ m/s } \left( \frac{5.0 \times 10^{-3}}{3.06} \right) \]

\[ = -0.49 \text{ m/s}. \]

(b) The mass of the man plus rifle is 74.5 kg. We use the same approach as in (a), to find

\[ v = -300 \text{ m/s } \left( \frac{5.0 \times 10^{-3}}{74.5} \right) \]

\[ = -2.0 \times 10^{-2} \text{ m/s}. \]
Chapter 6

**Momentum and Collisions**

6.1 Momentum and Impulse
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6.4 Glancing Collisions
6.3 Collisions

- **Inelastic collisions** – after two masses collide, they move together as one unit.

- **Characteristics of an inelastic collision:**
  - ✓ Momentum is conserved: \( \vec{p}_i = \vec{p}_f \)
  - ✓ Kinetic energy is not conserved: \( K_i \neq K_f \)
  - ✓ Before collision: \( \vec{p}_i = m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} \)
  - ✓ After collision: \( \vec{p}_f = (m_1 + m_2) \vec{v}_f \)
6.3 Collisions

Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision.

- Conservation of momentum becomes

\[
\begin{align*}
    m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} &= m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \\
    m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} &= (m_1 + m_2) \vec{v}_f
\end{align*}
\]
6.3 Collisions

- **Elastic collisions** – after two masses collide, they separate again.

- **Characteristics of an elastic collision:**
  - ✔ momentum is conserved
  - ✔ kinetic energy is conserved, also
6.3 Collisions

- Both momentum and kinetic energy are conserved.
- Typically have two unknowns

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]

\[
\frac{1}{2} m_1 \vec{v}_{1i} + \frac{1}{2} m_2 \vec{v}_{2i} = \frac{1}{2} m_1 \vec{v}_{1f} + \frac{1}{2} m_2 \vec{v}_{2f}
\]

- Solve the equations simultaneously
6.3 Collisions

Elastic Collisions

- A simpler equation can be used in place of the KE equation

\[ \vec{v}_{1i} - \vec{v}_{2i} = - \left( \vec{v}_{1f} - \vec{v}_{2f} \right) \]
6.3 Collisions

Summary of types of Collisions

- In an elastic collision, both momentum and kinetic energy are conserved.
- In an inelastic collision, momentum is conserved but kinetic energy is not.
- In a perfectly inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same.
Three carts of masses 4.0 kg, 10 kg, and 3.0 kg move on a frictionless horizontal track with speeds of 5.0 m/s, 3.0 m/s, and -4.0 m/s, as shown in the figure below. The carts stick together after colliding.

Questions:
(a) Find the final velocity of the three carts.
(b) Does your answer require that all carts collide and stick together at the same time?
Problems

Problem 6.3

Solution

(a) Using conservation of momentum $P_i = P_f$, we find

$$(4.0)(5.0) + (10)(3.0) + (3.0)(-4.0) = (4.0 + 10 + 3.0)v,$$

where $v$ is the speed of the three mass system after collision.

Therefore,

$v = +2.2 \text{ m/s}, \text{ or } 2.2 \text{ m/s toward the right.}$
**Problems**

◆ **Problem 6.3**

**Solution**

(b) No. For example, if the 10 kg and 3.0 kg mass were to stick together first, they would move with a speed given by solving

\[(13)v_1 = (10)(3.0) + (3.0)(-4.0),\]

So \(v_1 = 1.38 \text{ m/s}.\)

Then, when this 13 kg combined mass collides with the 4.0 kg mass, we have

\[(17)v = (4.0)(5.0) + (13)(1.38),\]

so that \(v = 2.2 \text{ m/s},\) just like part (a).
A 0.400-kg bead slides on a curved frictionless wire, starting from rest at point A, where $h=1.50$ m. At point $B$ the bead collides elastically with a 0.600-kg bead at rest.

**Questions:**
Find the distance the bead moves up the wire.
Problems

◆ Problem 6.4

Solution:
- We shall first use conservation of energy to find the speed of the bead just before it strikes the ball. The zero level of potential energy is at the level of point B. We have,

\[
\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f + 0 + (0.400 \text{ kg})(g)(1.50 \text{ m}) = \frac{1}{2}(0.400)v_{1f}^2 + 0,
\]

giving \(v_{1f} = 5.42 \text{ m/s}\).

- We now treat the collision to find the speed of the ball immediately after the collision. Momentum conservation gives:
Problems  
◆ Problem 6.4

Solution:
-Momentum conservation gives
\[(0.400 \text{ kg})(5.42 \text{ m/s}) + 0 = (0.400 \text{ kg})v_{1f} + (0.600 \text{ kg})v_{2f}\]
-For an elastic collision
\[v_{1f} + v_{1i} = v_{2f} + v_{2i} \Rightarrow v_{1f} + 5.42 = v_{2f} + 0\]
-Solving simultaneously both Eqs: \(v_{2f} = 4.34 \text{ m/s}\).
-Apply conservation of energy to the 0.6 kg ball after impact to find
\[\frac{1}{2}(0.600 \text{ kg})(4.34 \text{ m/s})^2 + 0 = 0 + (0.600 \text{ kg})(g)H,\]
or \(H = 0.96 \text{ m}\).
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- The “after” velocities have x and y components.
- Momentum is conserved in the x direction and in the y direction.
- Apply conservation of momentum separately to each direction.
Problems

◆ Problem 6.5

A 3.00-kg steel ball strikes a massive wall at 10.0 m/s at an angle of θ=60.0° with the plane of the wall. It bounces off the wall with the same speed and angle.

Questions:
If the ball is in contact with the wall for 0.200 s, what is the average force exerted by the wall on the ball?
Problems
◆ Problem 6.5

Solution:
Hints
-For x direction:
compute momentum =>
replace it in the formula

\[ \vec{F} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} \]

and find the force.
-Similarly for y direction
=> average force....
Chapter 6

Ends

Quiz 6 is coming ….. before Chapter 7

For review,
Study the following problems: 6.1 page 169, 6.2 page 170, 6.3 page 174, 6.4 page 177, 6.6 page 180, 6.7 page 181. Solve the following problems: 15, 16 page 191; 38 page 193, 46 page 194, 54 page 194.

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